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Vibrations of a plate on the Winkler basis, it is elastic support on a contour and keeping horizontal position in the neighborhood of the edge

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Abstract. *The are many examples of the plates suspended in various ways. Obviously expressed decisions as a rule are defined at balance of plates. At fluctuation of a plate frequency obviously analytically is not represented. In this work, we consider the oscillation of circular elastic plates on a Winkler base with complex attachment, the plate along the contour retains its horizontal position and rests elastically. Here the support stiffness is calculated as a function of frequency, the inverse problem is solved. The problem decision is represented in the form of Calvin's functions or Bessel. Results of calculation are presented in the form of schedules. At certain parities of cylindrical rigidity of a plate and frequency of free fluctuations the border of a qualitative picture of fluctuations takes place: on one party of this border of fluctuation Bessel's functions, on another - Calvin's functions are defined. The support stiffness is calculated here as a function of frequency, i.e. the inverse problem is solved.*

Keywords. oscillations · frequency · radius · stiffness · plates.

Mathematics Subject Classification (2010): 74K20

1 Introduction

Circular plates are widely applied in various branches of techniques as working elements in oil refining and the industries, aircraft engineering, in civil building, etc.

A number of works is devoted to definition of own frequencies of fluctuations of circular plates as free and based upon the elastic basis of type of Winkler [1-4]. Only for static problems about a bend of the rectangular plates lying on the elastic basis with variable

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factor of bed, decisions are known. In article [5] calculation of such plates is conducted by a method of final elements, and in [6] – a method of Galerkin.

In the given work free fluctuations of a round plate suspended with Winkler the basis are considered in the various ways. It will be obvious to affect a suspension bracket kind and frequency of fluctuations. In practice the plate support can appear distinct from planned and consequently the knowledge is necessary as it influences frequency of fluctuations. In work [7-10] symmetric cross-section fluctuations metallopolymer the three-layer circular plate connected with the elastic basis are investigated, at a heatstroke. For external layers Kirchhoff's hypotheses are accepted, in light filler the deformed normal is rectilinear and it is not squeezed on thickness. Analytical decisions are received their numerical analysis is carried out.

The decision of the equation of own fluctuations of the transversal - isotropic plate lying on the deformable basis is presented in work [11], one which the edge is rigidly fixed, and others three pivotally supported. The problem dares a method of decomposition, the frequency equation for definition of own cross-section fluctuations of a plate is received.

Problems of free fluctuations of round plates are considered at various variants of a suspension bracket in work [12].

At plate fluctuations on its support considerable efforts operate, as a result in which they are deformed and naturally it influences frequency of fluctuation of a plate.

2 Purpose of the study

Research of fluctuations of a plate in case of support pliability. To find out features of fluctuations in the presence of the basis of Winkler, namely in one case the decision is represented by means of functions of Bessel, in other by means Calvin's functions.

3 Statement a problem

The equation of fluctuations of a plate looks like [13]:

$$\varDelta\varDelta w + \frac{\kappa}{D} w + \frac{\rho h}{D} \cdot \frac{\partial^2 w}{\partial t^2} = 0.$$

Here substituting $w(r,t) = W(r)\cos\omega t$, if we turn to the dimensionless quantity and $\frac{k}{D} = 1$ if we accept it, then we will receive

$$\varDelta\varDelta W + \beta^4 W = W, \tag{3.1}$$

where

$$\left(\varDelta \pm \sqrt[4]{\beta^4 - 1}\right) \cdot W = 0,$$

where $\beta^4 = \frac{q\omega^2}{D}$, β- frequency functions; $D = \frac{Eh^3}{12(1-\nu)^2}$- cylindrical rigidity of a plate, E- the module of elasticity of a material of a plate; $q = \rho h$- the mass of a plate carried to surface unit, h- a thickness of a plate, ρ- material density; k- resistance of a ground to subsidence when the subsidence, carried to surface unit, is equal to unit; r_0- plate radius, ω-frequency of fluctuations, ν- coefficient of Puasson, $\varDelta$- operator Laplace.

The decision of the equation (3.1) looks like:

$$W = AZ_1\left(\sqrt[4]{\beta^4 - 1}\xi\right) + BZ_2\left(\sqrt[4]{\beta^4 - 1}\xi\right), \tag{3.2}$$

where $\xi = \frac{r}{\ell}$.

If

$$1)\quad \beta > 1;\quad W = AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right). \tag{3.3}$$

If

$$2)\quad \beta < 1;\quad W = A\cdot ber\left(0,\sqrt[4]{\beta^4-1}\xi\right) + B\cdot bei\left(0,\sqrt[4]{\beta^4-1}\xi\right). \tag{3.4}$$

Further derivatives $J_0\left(\sqrt[4]{\beta^4-1}\xi\right)$,$I_0\left(\sqrt[4]{\beta^4-1}\xi\right)$ Bessel and $ber\left(0,\sqrt[4]{\beta^4-1}\xi\right)$, $bei\left(0,\sqrt[4]{\beta^4-1}\xi\right)$ Calvin of functions will be necessary.

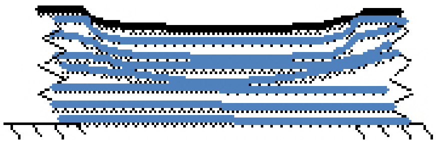

Fig.1. The plate near to edge keeps horizontal position and is elastic leans

The specified conditions look like:

$$\left.\frac{\partial W}{\partial \xi}\right|_{\xi=r_0} = 0;\quad D\left(\frac{\partial^3 W}{\partial \xi^3} + \frac{1}{\zeta}\cdot\frac{\partial^2 W}{\partial \zeta^2}\right) = \ell W. \tag{3.5}$$

Substituting (3.2) in (3.3), we have

$$\frac{\partial}{\partial \xi}\left(AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right) = 0$$

$$D\left(\frac{\partial^3\left(AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right)}{\partial \xi^3} + \right.$$

$$\left. + \frac{1}{\zeta}\cdot\frac{\partial^2\left(AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right)}{\partial \zeta^2}\right) =$$

$$= \ell\left(AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right) \tag{3.6}$$

$$AJ_1\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_1\left(\sqrt[4]{\beta^4-1}\xi\right) = 0 \tag{3.7}$$

$$D\left[A\left[\sqrt[4]{\beta^4-1}\left(\sqrt{\beta^4-1} - \frac{2}{\xi^2}\right)J_1\left(\sqrt[4]{\beta^4-1}\xi\right) + \frac{\sqrt{\beta^4-1}}{\xi}J_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right] + \right.$$

$$\left. + B\left[\sqrt[4]{\beta^4-1}\left(\sqrt{\beta^4-1} - \frac{2}{\xi^2}\right)I_1\left(\sqrt[4]{\beta^4-1}\xi\right) + \frac{\sqrt{\beta^4-1}}{\xi}I_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right]\right] =$$

$$= \ell\left(AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right). \tag{3.8}$$

Here $W = AJ_0\left(\sqrt[4]{\beta^4-1}\xi\right) + BI_0\left(\sqrt[4]{\beta^4-1}\xi\right) \quad \beta > 1.$

From (3.5)

$$A = -\frac{BI_1\left(\sqrt[4]{\beta^4-1}\xi\right)}{J_1\left(\sqrt[4]{\beta^4-1}\xi\right)}. \tag{3.9}$$

Substituting (3.7) in (3.6), we have

$$K(\beta) = \frac{\frac{\sqrt[4]{\beta^4-1}^2}{\xi}\left[J_0\left(\sqrt[4]{\beta^4-1}\xi\right)I_1\left(\sqrt[4]{\beta^4-1}\xi\right)-J_1\left(\sqrt[4]{\beta^4-1}\xi\right)I_0\left(\sqrt[4]{\beta^4-1}\xi\right)\right]}{J_0\left(\sqrt[4]{\beta^4-1}\xi\right)I_1\left(\sqrt[4]{\beta^4-1}\xi\right)-J_1\left(\sqrt[4]{\beta^4-1}\xi\right)I_0\left(\sqrt[4]{\beta^4-1}\xi\right)} \tag{3.10}$$

$$K(\beta) = \frac{\sqrt[4]{\beta^4-1}^2}{\xi}.$$

The schedule of function $K(\beta) - \beta$ of frequencies is presented on Fig.2.

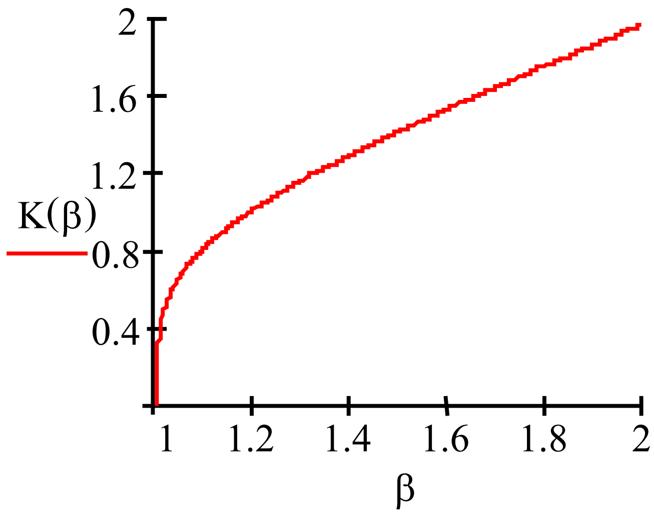

Fig.2. Presented function $K(\beta)$ $\beta > 1$ at the graph

Here $K(\beta) = \frac{\ell(\beta)}{D}$

If $\beta < 1$; $\quad W = A \cdot ber\left(0, \sqrt[4]{\beta^4-1}\xi\right) + B \cdot bei\left(0, \sqrt[4]{\beta^4-1}\xi\right)$

$$\frac{\partial}{\partial\xi}\left(Aber\left(0, \sqrt[4]{1-\beta^4}\xi\right) + Bbei\left(0, \sqrt[4]{1-\beta^4}\xi\right)\right) = 0$$

$$D\left(\frac{\partial^3\left(Aber\left(0, \sqrt[4]{1-\beta^4}\xi\right) + Bbei\left(0, \sqrt[4]{1-\beta^4}\xi\right)\right)}{\partial\xi^3} + \right.$$

$$\left. + \frac{1}{\zeta} \cdot \frac{\partial^2\left(Aber\left(0, \sqrt[4]{1-\beta^4}\xi\right) + Bbei\left(0, \sqrt[4]{1-\beta^4}\xi\right)\right)}{\partial\zeta^2}\right) =$$

$$= \ell\left(Aber\left(0, \sqrt[4]{1-\beta^4}\xi\right) + Bbei\left(0, \sqrt[4]{1-\beta^4}\xi\right)\right) \tag{3.11}$$

$$Aber\left(1,\sqrt[4]{1-\beta^4}\xi\right)+Bbei\left(1,\sqrt[4]{1-\beta^4}\xi\right)=0 \tag{3.12}$$

$$D\left[A\left[\sqrt[4]{1-\beta^4}\left(\sqrt{1-\beta^4}-\frac{2}{\xi^2}\right)ber\left(1,\sqrt[4]{1-\beta^4}\xi\right)+\frac{\sqrt{1-\beta^4}}{\xi}ber\left(0,\sqrt[4]{1-\beta^4}\xi\right)\right]+\right.$$

$$\left.+B\left[\sqrt[4]{1-\beta^4}\left(\sqrt{1-\beta^4}-\frac{2}{\xi^2}\right)bei\left(1,\sqrt[4]{1-\beta^4}\xi\right)+\frac{\sqrt{1-\beta^4}}{\xi}bei\left(0,\sqrt[4]{1-\beta^4}\xi\right)\right]\right]=$$

$$=\ell\left(Aber\left(0,\sqrt[4]{1-\beta^4}\xi\right)+Bbei\left(0\sqrt[4]{1-\beta^4}\xi\right)\right). \tag{3.13}$$

Here $W=Aber\left(0,\sqrt[4]{\beta^4-1}\xi\right)+Bbei\left(0,\sqrt[4]{\beta^4-1}\xi\right)$
From (3.12)

$$A=-\frac{Bbei\left(1,\sqrt[4]{\beta^4-1}\xi\right)}{ber\left(1,\sqrt[4]{\beta^4-1}\xi\right)}. \tag{3.14}$$

Substituting (3.12) in (3.11), we have

$$K(\beta)=\frac{\frac{\sqrt[4]{\beta^4-1}^2}{\xi}\left[ber\left(0,\sqrt[4]{1-\beta^4}\xi\right)bei\left(1,\sqrt[4]{1-\beta^4}\xi\right)-ber\left(1,\sqrt[4]{1-\beta^4}\xi\right)bei0\left(\sqrt[4]{1-\beta^4}\xi\right)\right]}{ber\left(0,\sqrt[4]{1-\beta^4}\xi\right)bei\left(1,\sqrt[4]{1-\beta^4}\xi\right)-ber\left(1,\sqrt[4]{\beta^4-1}\xi\right)bei0\left(\sqrt[4]{1-\beta^4}\xi\right)} \tag{3.15}$$

$$K(\beta)=\frac{\sqrt[4]{1-\beta^4}}{\xi}.$$

The schedule of function $K(\beta)-\beta$ of frequencies is presented on Fig. 3.
Here $K(\beta)=\frac{\ell(\beta)}{D}$.

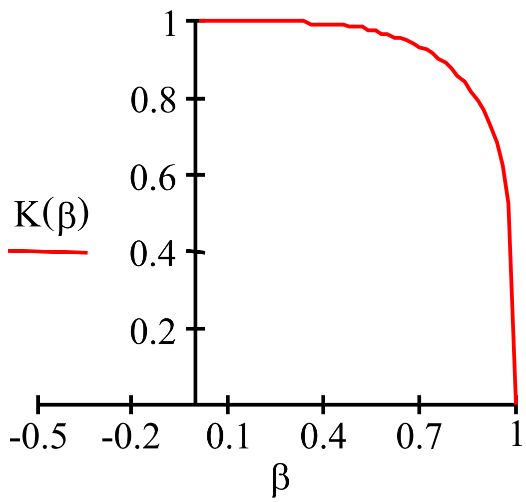

Fig.3. Presented function $K(\beta)$ $\beta>1$ at the graph

There are presented free fluctuations for various chances of fastening of round plates.

4 Conclusion

For the first time influence of a pliability of a support on fluctuations of elastic systems, in particular, on free fluctuations of a plate is studied. At that, in the presence of the elastic bases the interesting phenomenon is revealed: character fluctuation qualitatively depends on a parity of parameters of bed Winkler and elasticity of a support. The account of the studied phenomena is of interest for application in practice.

It is necessary to consider that the coefficient Winkler conducts to qualitatively changing condition of the fluctuations, characterized expression of decisions by various classes of function, namely Bessel's and Calvin's function.

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