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Wave flow of viscous fluid in elastic tube

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Abstract. *Although the main idea and principles of fluid flow in deformed pipes are known, it is theoretically actual the study of the more general law of the wave propagation processes in fluid flows through deformed tube. The results of investigations contain the base of wave process in shell-fluid systems. The mathematical model of the using system is described by the main equation of hydrodynamic. Solution of the problem is bringing to the singular problem for Sturm-Liouville equation.*

Keywords. pulsating flow · elastic tube · multiphase fluid · Sturm-Liouville equation.

Mathematics Subject Classification (2010): 76B55

1 Introduction.

The research of multiphase system dynamics covers a wide area of science, technology, living organisms and other fundamental problems.Although the main idea and principles of fluid flow in deformed pipes are known, the multiphase of fluid and the effect of various factors on the characteristic of fluid motion has not well-studied.Therefore, it is theoretically actual the study of the more general law of the wave propagation processes in fluid flows through deformed tube. The results of such investigations contain the base of the qualitative considerations of these or other facts characterize wave process in shell-fluid systems.

The mathematical model of the using system is described by the equation of incompressible viscous-elastic fluid motion, continuity equation and dynamic equation for linear-viscous elastic tube with changeable cross-section. Solution of the problem is bringing to the singular problem for Sturm-Liouville equation.

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2 Statement of the problem.

Incompressible viscous fluid flows in the semi-infinite tube with thickness h where the cross-section changes by law $R = R(x)$. Here $R(x)$ is monotone decreasing function for all $x \in [0, \infty), x$ is the coordinate has orientation along the tube axis. It consists of one-dimensional system of continuous hydro elastic equations [1 - 4]:

$$\frac{\partial}{\partial x}(Su) + L\frac{\partial w}{\partial t} = 0 \tag{2.1}$$

impulse equation:

$$\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(-p + \sigma). \tag{2.2}$$

And the motion equation of tube for the linear-viscous elasticity:

$$p = \frac{n}{R^2(x)}E^{\nu}w = \rho_* h\frac{\partial^2 w}{\partial t^2}. \tag{2.3}$$

When writing the last equation, it was considered that the tube was thin-walled and rigidly fixed to the environment. At the result the tube doesn't move along the axis. The classic description of viscous and ideal Newton fluid hydrodynamic is unacceptable while describing the full ambient flow with long molecular combinations. This fact posseses first degree importance for most technology processes that colloid substances, suspensions, emulsions and etc. include here.

To connect equations shown above, we write the rheology relations of fluid and accept it as linear viscous elastic.

$$\mathop{\mathit{\Pi}}_{j=1}^{r}\left(1 + \lambda_j\frac{\partial}{\partial t}\right)\cdot\sigma = 2\eta\mathop{\mathit{\Pi}}_{j=1}^{s}\left(1 + \theta_j\frac{\partial}{\partial t}\right)\cdot e. \tag{2.4}$$

In inequalities (2.1)-(2.4) $u(x,t)-$ is velocity of flow, $w(x,t)-$ radial displacement of tube walls, $p(x,t)-$ hydrodynamic pressure, $\sigma(x,t)$- tension, ρ and ρ_*- density of fluid and tube material, $e(x,t)-$ velocity of deformation, $S = \pi R^2$- area of cross-section, $L = 2\pi R(x)$- circular length of tube, η - dynamic viscous coefficient of fluid. λ_j and θ_j determine relaxation and retardation. Ln (2.3) E^{ν}- is inherited type operator [5].

$$E^{\nu} = E\left(1 - \mathit{\Gamma}^*\right), \mathit{\Gamma}^* w(x,t) = \int_{-\infty}^{l}\mathit{\Gamma}(t-\tau)w(x,\tau)d\tau$$

here $E-$ is the elasticity module, $\mathit{\Gamma}^*$ - is relaxation operator, $\mathit{\Gamma}(t-\tau)$ - is nuclear of relaxation. (2.3) is written openly as

$$p = \frac{h}{R^2(x)}E\left\{w(x,t) - \int_{-\infty}^{l}\mathit{\Gamma}(t-\tau)w(x,\tau)d\tau\right\}. \tag{2.5}$$

Take into account equality $e = \partial u/\partial x$ in (2.4):

$$\mathop{\mathit{\Pi}}_{j=1}^{r}\left(1 + \lambda_j\frac{\partial}{\partial t}\right)\cdot\sigma = 2\eta\mathop{\mathit{\Pi}}_{j=1}^{s}\left(1 + \theta_j\frac{\partial}{\partial t}\right)\cdot\frac{\partial u}{\partial x}. \tag{2.6}$$

Then the function $R(x)$ is written in the form $R(x) = R_\infty g(x)$, the function $g(x)$ is second-order differentiable. Here

$$\lim_{x\to\infty} g(x) = 1. \tag{2.7}$$

Simultaneously,

$$\lim_{x\to\infty} g'(x) = 0, \lim_{x\to\infty} g''(x) = 0. \tag{2.8}$$

Stokes show the differentiation with respect to x coordinate. This function can be shown as follows

$$g(x) = 1 + e^{-\beta x}\ (\beta > 0). \tag{2.9}$$

Expression (2.9) shows the narrowing of tube in the cone form according to the its length. Then taking into account the equations (2.5) and (2.6), we get the following closed system of equations:

$$\frac{\partial u}{\partial t} + 2\frac{g'(x)}{g(x)}u + \frac{2}{R_\infty g(x)}\frac{\partial w}{\partial t} = 0 \tag{2.10}$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma}{\partial x} \tag{2.11}$$

$$p = \frac{h}{R_\infty^2 g'(x)}E\left(w(x,t) - \int\limits_{-\infty}^{l} \Gamma(t-\tau)w(x,\tau)d\tau\right) \tag{2.12}$$

$$\mathop{\Pi}_{j=1}^{r}\left(\sigma + \lambda_j\frac{\partial \sigma}{\partial t}\right) = 2\eta \mathop{\Pi}_{j=1}^{s}\left(\frac{\partial u}{\partial x} + \theta_j\frac{\partial^2 u}{\partial x \partial t}\right). \tag{2.13}$$

Note that in wave processes the linearity of hydro-elasticity equations is true in the case that inequality $\left|u \cdot c^{-1}\right| << 1$ is hold:

$$\left|\frac{u}{c}\right| << 1,$$

c - is the complex propagation speed(for all times). The linearity of equations of theory of viscous elasticity is obtained from the kinematic impermeability.

3 Differential equation for velocity amplitude.

Let's convert the system of partial differential equations (2.10)-(2.13) to the system of Ordinary differential equations. Lets search solution in this form:

$$\begin{aligned} u &= u_1(x)\exp(i\omega t),\\ w &= w_1(x)\exp(i\omega t),\\ p &= p_1(x)\exp(i\omega t),\\ \sigma &= \sigma_1(x)\exp(i\omega t). \end{aligned} \tag{3.1}$$

Here u_1, w_1, p_1, σ_1- are complex functions of coordinates. The solution of corresponding problem is reduced to the solution of Storm-Louivelle problem by making some mathematical transformations. In conclusion, the following system of equations is obtained:

$$\begin{gathered} u(x,t) = \frac{Q_0}{\pi R_\infty^2 g(0)}\frac{F(x)}{F(0)}\exp(i\omega t),\\ w(x,t) = \frac{iQ_0}{\pi R_\infty \omega g^2(0) F(0)}\left\{\frac{1}{2}g(x)F'(x) + g'(x)F(x)\right\}\exp(i\omega t)\\ p(x,t) = iQ_0\frac{k(x)}{\pi R_\infty \omega g^2(0) F(0)}\left\{\frac{1}{2}g(x)F'(x) + g'(x)F(x)\right\}\exp(i\omega t),\\ \sigma(x,t) = 2Q_0\frac{\eta}{\pi R_\infty^2 g^2(0)}\frac{b}{a}\frac{F(x)}{F(0)}\exp(i\omega t). \end{gathered} \tag{3.2}$$

Here, the following substitutions and signes are used:

$$\alpha = \int_0^\infty \Gamma(\theta) e^{-i\omega\theta} d\theta; a = \prod_{j=1}^{r} (1 + i\lambda_j \omega); b = \prod_{j=1}^{s} (1 + i\theta_j \omega); F(x) = e^{-i\delta x}; \delta^2 =$$

$$= -\frac{i\omega\rho}{2\eta\frac{b}{a} - i\frac{R_\infty h}{2\omega}\left\{\frac{E}{R_\infty^2}(1-\alpha) - \rho_* \omega^2\right\}}; \xi = \theta/\lambda.$$

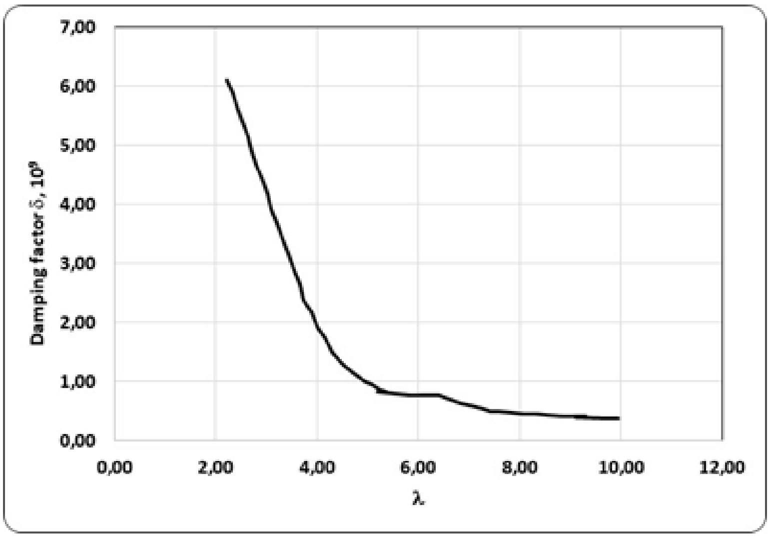

Fig.1. Dependence of damping factor from λ.

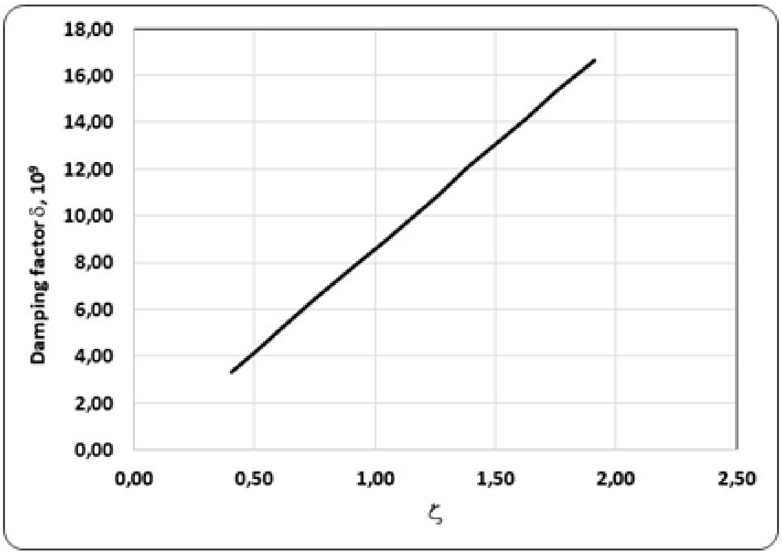

Fig.2. Dependence of damping factor from ξ ,$(\lambda = 5)$

4 Numerical report.

Let's show the dependence of reviewed fluids by experimenting them for define non-Newton properties. Following parameters are given:

$E = 4 \cdot 10^6 dN/cm^2$, $\rho = 1\ gr/cm^3$, $\rho_* = 1 gr/cm^3$, $R = 1,2\ cm$, *h=0,2cm,*

$Q_0 = 120\ cm^3/\sec$, $\omega = 2\pi \sec^{-1}$, *x=10cm,* $\eta = 5gr/cm \cdot \sec$, $\eta = 14$

Calculations show that, the propagation of wave speed for accepted model almost doesn't depend on λ and ξ , and equals to $577cm/sec$.

In Fig. 1 and 2 the dependence of damping factor from λ and ξ is given.

References

1. Aliev, A.B.: *On the motion of waves in multilayer elastic tube, containing visco-elastic fluid* , Science and World International Scientific journal Volgograd, (12), 10–12 (2017).
2. Aliev, A.B.: *Waves in the liquid proceeding in the elastic tube considering viscoe-lastic friction of the environment* Science and World International Scientific journal , Volgograd, (12), 8–10 (2017).
3. Amenzade, R.Yu.: *Volny v neodnorodnoy trubke s protekayushchey dvuhfaznoy zhidkost'yu* , Mekhanika kompozitnyh materialov, **39** (4), 555–566 (2003).
4. Amenzade, R.Yu., Ahundov, M.B., Kiyasbejli, E.T.: *Pul'siruyushchee techenie maksvellovskoj zhidkosti v vyazkouprugoy trubke* Baki Universitetinin Xeberleri (2), 138–145 (2002).
5. Pedli, T.Zh.: Gidrodinamika krupnyh krovenosnyh sosudov, *M.: Mir,* 400 p. (1983).
6. Rabotnov, Yu.N.: Elementy nasledstvennyj mekhaniki tverdyh tel, *M.: Nauka,* 382 p. (1977).
7. Volmir, A.S.: Obolochki v potoke zhidkosti i gaza. Zadacha uprugosti. *M.: Nauka*, 320 p. (1979).