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Scattering of the internally reinforced damaged pipe in a cylindrical form with active material

Yusif M. Sevdimaliyev · Sahib A. Piriev ·
Esmira M. Mustafayeva

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Abstract. *The article deals with the problem of determining the long-term strength of a damaged pipe that is internally in contact with an active cylindrical layer. The contact of the pipe with the active substance leads to a decrease in the strength characteristics. The equation of the destruction front is obtained and analyzed.*

Keywords. aggressive environment · concentration of aggressive environment · stress intensity · damageability

Mathematics Subject Classification (2010): 74A45

1 Introduction

Experiments on the resistance of deformed structural elements in contact with aggressive media show that aggressive media have a significant softening effect on the mechanical properties of construction materials. This led to the need to reassess the calculations for strength and durability in similar situations. One of the ways to study this issue is the structural-phenomenological approach. This approach was demonstrated in [2]. In this paper, using the results of [4,6], the problem of determining the long-term strength of a perpendicular cylindrical tube, internally reinforced by a coaxial cylindrical layer of active material, is solved. In the model used, the influence of the active medium is related to the penetration of the components of the medium into the body due to the diffusion process.

A quantitative measure of the degree of the presence of a substance in the medium in a body is the concentration in it of the components of this substance.

Y.M. Sevdimaliyev
Baku State University, Baku, Azerbaijan
E-mail: yusifsev@mail.ru

S.A. Piriev
Baku State University, Baku, Azerbaijan
E-mail: piriyev@bsu.edu.az

E.M. Mustafayeva
Azerbaijan Economic University (UNEC), Baku, Azerbaijan
E-mail: esi.m87@mail.ru

2 Problem statement

As an equation characterizing the distribution of the concentration of an aggressive medium in the body, the diffusion equation is adopted:

$$\frac{\partial C}{\partial t} = div\,(D\,grad\,C) \tag{2.1}$$

with a zero initial value of the concentration of the aggressive substance in the body and the boundary condition

$$C\,(\mathbf{r},\ t)\,|_{\mathbf{r}\in s} = 1, \tag{2.2}$$

where $\overrightarrow{r}$ - the vector coordinate of the body point, S- the surface of the body, C- the concentration of components of the aggressive medium at a given point of the body, referring to its value on the surface of the body boundary.

As in [3], we will assume that the properties of structural elements depend on the presence of environmental components in the body, which manifests itself in decreasing the short-time strength limit of the body's main material.

At a certain level of loading of the body, it begins to gradually break down. In this regard, the external load is redistributed between the remaining unauthorized parts of the body. The boundary of the expanding fractured region of the body represents the destruction front, the propagation velocity of which determines the durability or long-term strength.

The level of the stressed state of a structural element is characterized by an equivalent voltage σ_E, in the quality of which the stress intensity is taken in this work. As a condition for the destruction of a structural element that is in contact with an aggressive medium, let us accept the condition for reaching this limit of short-term strength in the presence of a medium:

$$\sigma_E = \sigma_\Pi. \tag{2.3}$$

However, due to the damaging nature of the material of the body, failure will occur at a lower load. According to the hereditary theory of damageability [6]-[8], the criterion of destruction will look like this:

$$(1 + M^*)\,\sigma_E = \sigma_\Pi, \tag{2.4}$$

where * is the integral damage operator.

The limit of short-term strength $\sigma$is a function of the concentration of an aggressive substance in the body. In this paper, a linear approximation of this dependence is adopted:

$$\sigma_\Pi\,(C) = \sigma_{\Pi 0}\,(1 - \gamma\,C)\,, \tag{2.5}$$

where $0 < \gamma < 1$ is the empirical constant. The focus of destruction occurs at some point in time - called the incubation period, at the point of the body where condition (2.5) is first satisfied. After that, the front of destruction appears in the body and begins to spread. The body collapses when either the velocity of the fracture front turns to infinity, or when the destructive part covers the whole body.

A more precise approach to this problem is based on taking into account the presence of residual strength behind the fracture front, when the material of the body behind the fracture front retains to some extent the bearing capacity. In this paper, this approach is realized in the following variant: it is assumed that when the condition (2.5) is satisfied, the material loses the ability to accumulate damages, instantaneous qualitative restructuring of the structure takes place, so that its behavior can be described by a model perfectly elastic body, but with sharply reduced values of the stiffness parameters of the Young's elastic moduli of shear.

Suppose that for the problem under consideration the conditions ensuring the state of plane deformation are satisfied. Then the design is represented to a sufficient degree by its cross-section, representing a concentric ring of outer radius b, internal - a and radius δ - contact of the internal aggressive environment $S_0$and outer area - $S_1$the main body. At the boundary $r = a$, a uniformly distributed pressure p is given (Fig. 1).

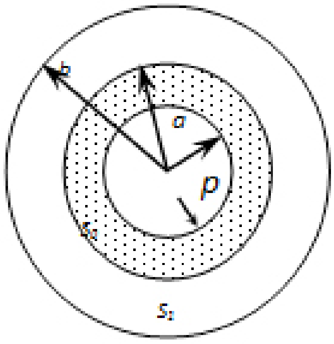

Fig. 1. Cross section of the structure

The work [5] is devoted to the study of this problem until the appearance of a fracture nucleus. In this paper, we investigate the further process of destruction associated with the motion of the destruction front. In this case, the outer annular region of the body is divided into two annular regions S_p and S_1, whereS_1 is the region of the undisturbed part of the body, the region of the body behind the destruction front with power strength and stiffness characteristics (Fig. 2).

Denote by d - the variable radius of the fracture front $d = d(t)$.

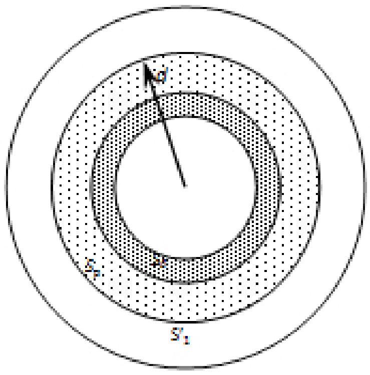

Fig. 2. Diagram of the location of the destruction front.

Also denote by q_1- the contact pressure on the contact surface $r = \delta$, through q_2- the contact pressure on the front of failure at $r = d$. For simplicity, we shall consider the materials of all three considered regions to be incompressible. Then the stresses in these regions will be written as:

in area $S_0 : (a < r < \delta) :$

$$\sigma_r^{(c)} = \frac{pa^2 - q_1\delta^2}{\delta^2 - a^2} - \frac{(p - q_1)\delta^2 a^2}{(\delta^2 - a^2)r^2}; \tag{2.6}$$

$$\sigma_\theta^{(c)} = \frac{pa^2 - q_1\delta^2}{\delta^2 - a^2} + \frac{(p - q_1)\delta^2 a^2}{(\delta^2 - a^2)r^2}; \tag{2.7}$$

in area $S_1 : (d < r < b)$:

$$\sigma_r^{(e)} = \frac{q_2 d^2}{b^2 - d^2}\left(1 - \frac{b^2}{r^2}\right) \tag{2.8}$$

$$\sigma_\theta^{(e)} = \frac{q_2 d^2}{b^2 - d^2}\left(1 + \frac{b^2}{r^2}\right) \tag{2.9}$$

The axial normal stresses for planar deformation are determined as follows:

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) \tag{2.10}$$

For simplicity, the pipe material is considered incompressible, i.e. Poisson's ratio $\nu = 0,5$. Then formula (2.8) takes the following form:

$$\sigma_z = 0,5(\sigma_r + \sigma_\theta) \tag{2.11}$$

The intensity of stresses or the resulting stress is determined by the formula:

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_\theta - \sigma_z)^2} \tag{2.12}$$

Substituting the values of (2.9) into (2.10), we find

$$\sigma_i = \pm\frac{\sqrt{3}}{2}(\sigma_r - \sigma_\theta). \tag{2.13}$$

Substituting formula (2.11) into formula (2.4), we obtain:

$$\sigma_\theta - \sigma_r = \frac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi \tag{2.14}$$

The equation of equilibrium can be written in the form [3]:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2.15}$$

Substituting formula (2.12) into formula (2.13), we obtain the stress component in the destroyed zone:

$$\begin{aligned} \sigma_r^{(p)} &= -q_1 + \tfrac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi(\tau)\ln\tfrac{r(\tau)}{\delta(\tau)} \\ \sigma_\theta^{(p)} &= -q_1 + \tfrac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi(\tau)\left(1 + \ln\tfrac{r(\tau)}{\delta(\tau)}\right) \end{aligned} \tag{2.16}$$

From the condition of continuity of radial and tangential stresses on the contact surface using formulas (2.7) and (2.14), we obtain the following two equations:

$$\begin{cases} -\frac{qd^2(t)}{b^2 - d^2(t)}\left(\frac{b^2}{d^2(t)} - 1\right) = -q_1 + \frac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi(\tau)\ln\frac{d(\tau)}{\delta(\tau)}; \\ \frac{qd^2(t)}{b^2 - d^2(t)}\left(\frac{b^2}{d^2(t)} + 1\right) = -q_1 + \frac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi(\tau)\left(1 + \ln\frac{d(\tau)}{\delta(\tau)}\right) \end{cases} \tag{2.17}$$

From the first equation of (2.15) we find

$$q_2 = q_1 - \frac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi(\tau)\ln\frac{d(\tau)}{\delta(\tau)} \tag{2.18}$$

and substituting in the second equation (2.15), we obtain

$$q_2 \cdot \frac{b^2 + d^2}{b^2 - d^2} = -q_1 + \frac{2}{\sqrt{3}}(1 - M^*)\sigma_\Pi(\tau)\left(1 + \ln\frac{d(\tau)}{\delta(\tau)}\right) \tag{2.19}$$

From the condition of continuity of the tangential stresses on the contact surface with the use of formulas (2.6) and (2.14), we obtain the following equation:

$$q_1 = p - \frac{1}{\sqrt{3}}(1 - M^*)\frac{\delta^2(\tau) - a^2}{a^2} \cdot \sigma_{\Pi}(\tau) \tag{2.20}$$

Thus, we have a system of three Volterra integral equations of the second kind (2.17), (2.18) and (2.19) with respect to three unknown functions: the contact pressures $q_1\,(t)$ and $q_2\,(t)$ the radial coordinate of the destruction front.

It should be noted that in these equations the $d\,(t)$ function has the following structure:

$$d\,(t) = \begin{cases} a; & t \le t_0 \\ d\,(t)\,; & t > t_0 \end{cases}$$

Where t_0 is the incubation time, that is, the time of occurrence of the foci of destruction on the contact surface $r = b$.

According to formulas (2.17) and (2.19) it follows that for certain relations of the magnitude of the internal pressure p, the strength σ_0 of a defect-free material, the ratio of geometric dimensions, detachment can occur both on the contact surface $r = b$and on the fracture front $r = d$. The moment of detachment is determined by the conditions $q_1\,(t) \le 0$ and$q_2\,(t) \le 0$. However, a comparison of formulas (2.18) and (2.20) shows that $q_2\,(t) < q_1\,(t)$. This means that the detachment will first take place at the front of destruction.

Then the solution of equation (2.18) with allowance for (2.17) is valid until the moment of detachment, that is, if the $q_2\,(t) > 0$condition is satisfied.
So there are two possible ways of breaking the pipe: 1) because of detachment, the condition $q_2 = 0$ is fulfilled; 2) due to scattered destruction, when the fracture front reaches the outer boundary $r = b$ and the condition $q_2\,(t) > 0$ is always satisfied.

In the problem under consideration, for the concentration of the active substance in the tube, the boundary conditions are taken in the form [7]

$$C(a;t) = 1,\ C(b;t) = 0 \tag{2.21}$$

We assume that the quasi-stationary distribution of the concentration of the active substance of the type takes place at the tube thickness:

$$C = \frac{b - d}{b - a} \tag{2.22}$$

The process of scattered fracture is investigated according to the scheme of Lazar Kachanov [1]. Destruction, starting at the inner boundary of the pipe, where the intensity of stresses is maximum, develops to the outside. To determine the law of motion of the front, we introduce the following dimensionless quantities:

$$\frac{d}{b} = \beta;\ \frac{\delta}{b} = \beta_0;\ \frac{\delta}{a} = k;\ \frac{q_2}{p} = \tilde{q}_2;\ \frac{d}{\delta} = \frac{\beta}{\beta_0};\ \frac{q_1}{p} = \tilde{q}_1;\ g = \frac{\sigma_{T_0}}{\sqrt{3}p}.0 < \tau < t, \tag{2.23}$$

and:

$$M(\tau) = mK(\tau);\quad m\tau = \varsigma;\ mt = s \tag{2.24}$$

Then, taking into account (2.22) and (2.23), from (2.18) we obtain the following nonlinear integral equation with respect to the dimensionless radius of the destruction front $\beta(t)$:

$$\tilde{q}_2 \cdot \frac{1 + \beta^2(s)}{1 - \beta^2(s)} = -\tilde{q}_1 + 2g\left(1 - \gamma \cdot \frac{1 - \beta(s)}{1 - k} - \int_0^t K(\varsigma)\left(1 - \gamma \cdot \frac{1 - \beta(\varsigma)}{1 - k}\right)\right) \tag{2.25}$$

Numerical realization was carried out for three types of cores of the operator of damageability [8]: singular $M(t) = mt^{-\alpha}, \quad 0 < \alpha < 1$, $M(t) = me^{-\alpha t}$, and constant $M(t) = m$ for the initial relative width of the pipe $\beta_0 = 0,5$.

Figures 3-4 show the fracture front curves based on numerical calculation data.

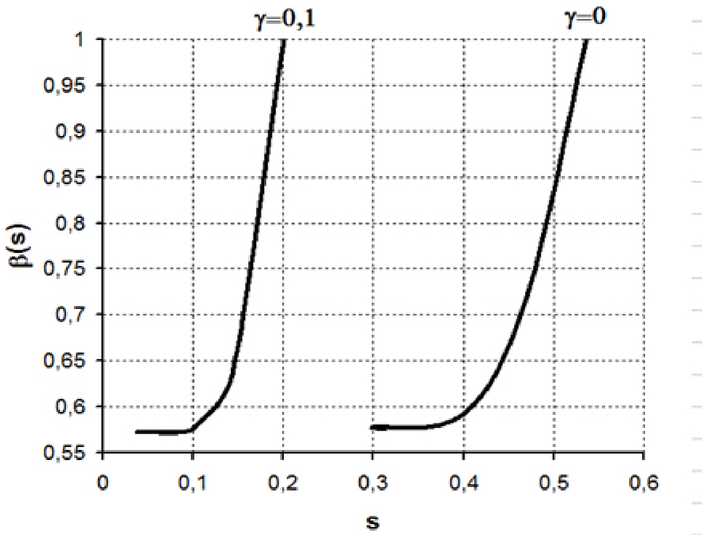

Fig. 3. Curves of the fracture front for the nucleus (k = 1.4).

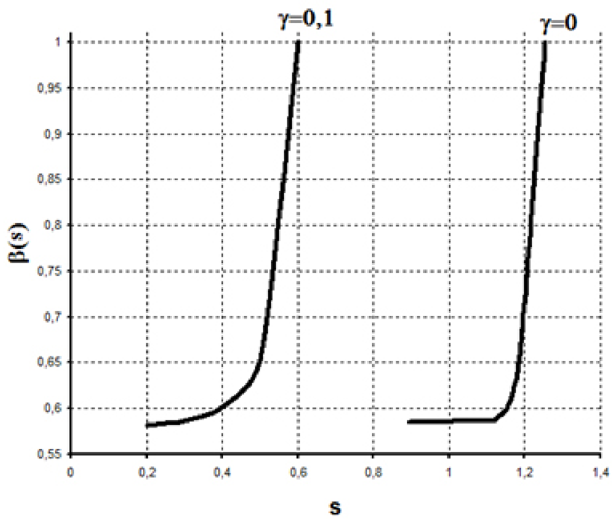

Fig. 4. Curves of the fracture front for the nucleus (k = 1,2).

3 The conclusion

The integral equation is derived with respect to the radial coordinate of the fracture front, taking into account the diffusion processes on the contact surface of the pipe with the active filler, as well as the damage process of the material of the pipe itself. Explicit formulas are obtained for the contact pressures at the fracture front and the surface of adhesion of the pipe to the active substance. The analysis of the relationship between the critical situations of delamination on the contact surface of a pipe with a filler, as well as on the front of fracture and analysis of fracture due to the accumulation of a critical volume of damage is analyzed.

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