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On the problems of tension, twisting and bending of ropes

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Abstract. *The paper gives a brief description of the construction of a new theory of the rope, based on the spectral theory operators. The ropes are modeled as a cylinder with a helical anisotropy, in which the filler is eliminated by means of the limiting transition in the material characteristics obtained on the basis of the averaging theory. Based on the solutions of the Saint-Venant problems of tension-torsion and bending of ropes, new formulas for calculating the elements of the stiffness matrix are obtained. The calculation results are presented graphically.*

Keywords. helical anisotropy · rope · averaging theory · Saint-Venant problem, rigidity · stiffness matrix

Mathematics Subject Classification (2010): 74B05

1 Introduction

There are different designs of ropes, which differ mainly in the way they are braided and the profile of the wire cross-section from which they are twisted [4]. Such diversity is caused by different conditions of their exploitation. The most common ongoing are single and double lay round steel ropes. For single-wire ropes, the wires (fibers) are arranged along the twisting spirals around the central rectilinear fiber in several layers. Two lay rope are woven from lay. There are two basic approaches to constructing the theory of single lay rope (the lay of an ordinary rope will refer as a ”rope”). One such approach [4, 2] is based on the concept of a rope as a discrete system of curvilinear rods and uses the methods of construction mechanics. The second approach is based on the equations of an elastic continuous medium with helical anisotropy [6, 7, 9].

Unlike rectilinear rods, where all the known approaches to constructing an elementary theory (the method of hypotheses, the Saint-Venant theory, the asymptotic methods of the theory of elasticity) give the same result

$$B_r = d_{11} = ES$$

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where B_r is the tensile rigidity, E is the Young's modulus, S is the cross-sectional area, in the rope theories different approaches lead to different analytical expressions for the elements of the stiffness matrix d_{ij}.

Adduce the expressions d_{11}, obtained by different authors, to illustrate the variety of the available formulas,

Gegauff (1907, by methods of the theory of elasticity)

$$B_r = ES_0 \cos^2(\alpha)$$

where S_0 is the effective cross-sectional area of the rope, α is the angle between the tangent to the outermost fiber and the axis of the rope.

Dinnik A.N. (1957, by the methods of construction mechanics)

$$B_r = ES_0 \cos^4(\alpha)$$

Glushko M.F. (1966, by the methods of construction mechanics)

$$B_r = \sum_{i=1}^{m} (ES_i \cos^3(\alpha_i) + 1/r_i^2 EI_{pi} \sin^4(\alpha_i) \cos^3(\alpha_i) + 1/r_i^2 GI_{pi} \sin^6(\alpha_i) \cos^2(\alpha_i))$$

there m is the number of fibers in the rope, G is the shear modulus, S_i, I_i, I_{pi} - cross sectional area section, the moment of inertia of the section with respect to the axis lying in the transverse section, the polar moment of inertia of the i-th fiber, respectively, r_i - the distance between the axis of the rope and the fiber, α_i is the angle of inclination of the fiber to the axis of the rope. The construction of the stiffness matrix was carried out in [3] on the basis of the Saint-Venant solution for a cylinder of a fiber composite with a weak filler. Since the construction of the solution is related to the numerical integration of a second-order differential equation whose coefficient with the highest derivative tends to zero as the aggregate's Young modulus tends to zero. This process of numerical integration is unstable and does not allow to obtain an "exact result" in the limiting case. Therefore, the stress-strain state (SSS) calculations were duplicated by constructing the solution of the three-dimensional problem of the theory of elasticity for a continuous inhomogeneous cylinder formed by a finite number of elastic helical spirals connected by a weak filler by the finite element method (FEM). In the present paper, a new approach is used to determine the SSS and its rigidity d_{ij}, which allows to obtain expressions for the elements of the stiffness matrix in the form of elementary functions of the parameter α.

In [3], the stiffness matrix was constructed on the basis of helical spirals so that the step h of each of them and the twist $\tau = 2\pi h$ remain constant. Simultaneously with the winding the layers are coated with a polymer binder. After polymerization of the bonding layers we obtain a cylinder of a fibrous composite.

We denote by E_1, ν_1 the Young's modulus and the Poisson's ratio of helix; through E_2, ν_2, the elastic characteristics of the aggregate. To describe the integral elastic properties of such a cylinder, we proceed as follows: with the geometric center of gravity of one of the ends of the cylinder we connect the origin of the Cartesian coordinate system x_1, x_2, x_3. This coordinate system will be called the base coordinate system. We introduce the helical coordinate system r, θ, z, associated with the basic relations

$$x_1 = rcos(\theta + \tau z), \quad x_2 = rsin(\theta + \tau z), \quad x_3 = z. \tag{1.1}$$

The relations (1.1) for $r = const, \theta = const$ are the parametric equations of the helical fiber.

The radius vector of fiber points (lines) is represented in the form

$$\mathbf{R} = r\mathbf{e_1'} + z\mathbf{e_3'}, \tag{1.2}$$

There

$$\mathbf{e_1'} = \mathbf{e_r} = \mathbf{i_1} cos(\theta + \tau z) + \mathbf{i_2} sin(\theta + \tau z),$$
$$\mathbf{e_2'} = \mathbf{e_\theta} = -\mathbf{i_1} sin(\theta + \tau z) + \mathbf{i_2} cos(\theta + \tau z)$$

there $\mathbf{i_1}, \mathbf{i_2}, \mathbf{i_3}$ are the unit vectors of the basic (Cartesian) coordinate system. With a helical line we connect the natural basis (Frenet frame)$\mathbf{e_1} = \mathbf{n}$, $\mathbf{e_2} = \mathbf{b}$, $\mathbf{e_3} = \mathbf{t}$ the units of the principal normal, binormal and tangent, respectively.

The orthogonal transition matrix from the basis $\mathbf{e_j}$ to the basis $\mathbf{e'_i}$ has the form

$$A = \left\| \begin{matrix} -1 & 0 & 0 \\ 0 & -\cos\beta & \sin\beta \\ 0 & \sin\beta & \cos\beta \end{matrix} \right\|$$

where h

$$\beta = arctg(x), \;\; x = r\tau$$

The cylinder material obtained by the method described above is not homogeneous, however, for a sufficiently large number of winding layers it can be considered as locally transversally isotropic at each point of the cylinder based on the averaging theory, with the direction of the principal symmetry axis coinciding with the vector $\mathbf{e_3} = \mathbf{t}$ [6] .

To describe the elastic properties of a cylinder, we use the vector-matrix form of the generalized Hooke's law [10]:

$$\begin{array}{l} \sigma = \mathbf{Ce} \\ \mathbf{e} = [e_1, ..., e_6]^T, \;\; \sigma = [\sigma_1, ..., \sigma_6]^T, \;\; \mathbf{C} = (c_{ij}), i, j = 1, ...6 \\ \sigma_1 = \sigma_{11}, \;\; \sigma_2 = \sigma_{22}, \;\; \sigma_3 = \sigma_{33}, \;\; \sigma_4 = \sigma_{23}, \;\; \sigma_5 = \sigma_{13}, \;\; \sigma_6 = \sigma_{12} \\ e_1 = e_{11}, \;\; e_2 = e_{22}, \;\; e_3 = e_{33}, \;\; e_4 = 2e_{23}, \;\; e_5 = 2e_{13}, \;\; e_6 = 2e_{12} \end{array}$$

there σ_{ij}, e_{ij} are the components of stress and deformation tensors, respectively.

The elastic properties of a transversely isotropic material are determined by five technical constants: Young's modulus E, E', Poisson's coefficients ν, ν' and the shear modulus G'. Elements of the matrix $\mathbf{C}$ are expressed in terms of these constants formulas

$$\begin{array}{l} c_{11} = c_{22} = \dfrac{E(E' - E\nu'^2)}{E'(1-\nu^2) - 2E\nu'^2(1+\nu)} \;\; c_{12} = \dfrac{E(\nu E' + \nu'^2 E\nu^2)}{E'(1-\nu^2) - 2E\nu'^2(1+\nu)}, \\ c_{13} = c_{23} = \dfrac{EE'\nu'}{E'(1-\nu) - 2\nu'^2 E}, \;\; c_{33} = \dfrac{E'^2(1-\nu)}{E'(1-\nu) - 2\nu'^2 E}, \\ c_{44} = c_{55} = G'; \;\; c_{66} = \dfrac{E}{2(1+\nu)} \;\; c_{15} = c_{16} = c_{25} = c_{26} = c_{35} = c_{36} = 0 \end{array}$$

Based on the averaging theory [6], we have

$$\begin{array}{l} E = (\dfrac{k_1(1-\nu_1^2)}{E_1} + \dfrac{k_2(1-\nu_2^2)}{E_2} + \dfrac{\nu'^2}{E'})^{-1} \\ \nu = E[\dfrac{k_1(\nu_1+\nu_1^2)}{E_1} + \dfrac{k_2(\nu_2+\nu_2^2)}{E_2} - \dfrac{\nu'^2}{E'}] \\ G = [\dfrac{2k_1(1+\nu_1)}{E_1} + \dfrac{2k_2(1+\nu_2)}{E_2}]^{-1} \\ E' = k_1 E_1 + k_2 E_2, \;\; \nu' = k_1\nu_1 + k_2\nu_2, \;\; k_1 + k_2 = 1 \end{array}$$

where k_1, k_2 is the concentration of the bearing elements and the filler respectively along the section perpendicular to $\mathbf{e_3}$

As a result of the transition from the basis $\mathbf{e_j}$ to the basis $\mathbf{e'_i}$, we obtain the following relations of the generalized Hooke's law in a helical coordinate system

$$\begin{array}{l}
\sigma_{rr} = c'_{11}e_{rr} + c'_{12}e_{\theta\theta} + c'_{13}e_{zz} + 2c'_{14}e_{\theta z} \\
\sigma_{\theta\theta} = c'_{12}e_{rr} + c'_{22}e_{\theta\theta} + c'_{23}e_{zz} + 2c'_{24}e_{\theta z} \\
\sigma_{zz} = c'_{13}e_{rr} + c'_{23}e_{\theta\theta} + c'_{33}e_{zz} + 2c'_{34}e_{\theta z} \\
\sigma_{\theta z} = c'_{14}e_{rr} + c'_{24}e_{\theta\theta} + c'_{34}e_{zz} + 2c'_{44}e_{\theta z} \\
\sigma_{rz} = c'_{55}e_{rz} + c'_{56}e_{r\theta} \\
\sigma_{r\theta} = c'_{56}e_{rz} + c'_{66}e_{r\theta}
\end{array} \tag{1.3}$$

There

$$\begin{array}{l}
c'_{11} = c_{11},\ \ c'_{12} = c_{12}l_c^2 + c_{13}l_s^2,\ \ c'_{13} = c_{13}l_c^2 + c_{12}l_s^2, \\
c'_{14} = l_c l_s(c_{13} - c_{12}), \\
c'_{23} = c_{13}l_c^2 + (c_{11} + c_{33} - 4c_{44})l_c^2 l_s^2 + c_{13}l_s^4 \\
c'_{24} = -c_{11}l_c^3 l_s - c_{13}(l_c l_s^3 - l_c^3 l_s) + c_{33}l_c l_s^3 - 2c_{44}(l_c l_s^3 - l_c^3 l_s) \\
c'_{33} = c_{11}l_s^4 + 2c_{13}l_c^2 l_s^2 + c_{33}l_c^4 + 4c_{44}l_c^2 l_s^2 \\
c'_{34} = -l_c l_s(c_{11}l_s^2 - c_{13} + 2c_{13}l_c^2 + 2c_{44}l_c^2 - 2c_{44}l_s^2) \\
c'_{44} = c_{11}l_c^2 l_s^2 - 2c_{13}l_c^2 l_s^2 + c_{33}l_c^2 l_s^2 + c_{44}(1 - 4l_c^2 l_s^2) \\
c'_{55} = c_{44}l_c^2 + c_{66}l_s^2, \\
c'_{56} = l_c l_s(c_{44} - c_{66}) \\
c'_{66} = c_{66}l_c^2 + c_{44}l_s^2 \\
l_c = cos\beta,\ \ l_s = sin\beta,\ \ \beta = arctg(\tau \mathrm{r})
\end{array} \tag{1.4}$$

there $r \in [0, a]$, a - is the outer radius of the rope.

The components of the deformation tensor in the basis of the helical coordinate system are expressed in terms of the displacements by the following relations

$$\begin{array}{ll}
e_{rr} = \partial_r u_r, & e_{\theta\theta} = u_r + \partial_\theta u_\theta / r, \\
e_{zz} = D u_z, & 2e_{r\theta} = \partial_r u_\theta + (\partial_\theta u_r - u_\theta)/r, \\
2e_{\theta z} = (\partial_\theta u_z)/r + D u_\theta, & 2e_{rz} = \partial_r u_z + D u_r,\ \ D = \partial_z - \tau\partial_\theta
\end{array} \tag{1.5}$$

Equations of equilibrium in stresses in this case have the form

$$\begin{array}{l}
\dfrac{\partial\sigma_{rr}}{\partial r} - \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \dfrac{1}{r}\dfrac{\partial\sigma_{r\theta}}{\partial\theta} + D\sigma_{rz} = 0 \\
\dfrac{\partial\sigma_{r\theta}}{\partial r} + 2\dfrac{\sigma_{r\theta}}{r} + \dfrac{1}{r}\dfrac{\partial\sigma_{\theta\theta}}{\partial\theta} + D\sigma_{\theta z} = 0 \\
\dfrac{\partial\sigma_{rz}}{\partial r} + \dfrac{\sigma_{rz}}{r} + \dfrac{1}{r}\dfrac{\partial\sigma_{\theta z}}{\partial\theta} + D\sigma_{zz} = 0
\end{array} \tag{1.6}$$

Using relations (1.3)-(1.6), one can obtain a system of differential equations with respect to the components of the displacement vector, $\mathbf{u} = [u_r, u_\theta, u_z]^T$, which we symbolically write in the form

$$L(\partial)\mathbf{u} = 0,\ \ \partial = \frac{\partial}{\partial z} \tag{1.7}$$

The condition for the absence of stresses on the lateral surface is written in the form

$$N(\partial)\mathbf{u} = 0 \tag{1.8}$$

If we look for a solution in the form

$$\mathbf{u} = \mathbf{a}(r,\theta)e^{i\lambda z} \tag{1.9}$$

after substituting (1.7) into (1.5), (1.6) we obtain the spectral problem on the cross section

$$Z(i\lambda)\mathbf{a} = \{L(i\lambda)\mathbf{a}, N(i\lambda)\mathbf{a}\} = 0$$

The operator Z has an infinite countable set of eigenvalues $\lambda_k^{\pm}$, among which $\lambda_0 = 0, \lambda_1^{\pm} = \pm\tau$ are quadruple. Linear combinations of the eigenvectors and associated vectors of eigenvalues determine six trivial elementary solutions describing the rope's displacement as a rigid body, three Saint Venant solutions of tensile-torsion problems, pure bending and bending of transverse forces and trivial elementary solutions linear combination of which describes various displacements rope as a solid body. The remaining eigenvalues are complex and symmetrically located in the complex plane [3,6,8]. These properties allows to present a general solution for the rope in the following form [1,8]:

$$\mathbf{u} = \sum_{n=1}^{12} C_n \mathbf{u}_n + \sum_k [C_k^+ \mathbf{u}(z, \lambda_k^+) + C_k^- \mathbf{u}(z - l, \lambda_k^-)]$$

Here, l is the length of the cylinder $C_n, C_k^{\pm}$ are arbitrary constants that are determined by satisfying the boundary conditions at the ends of the cylinder $z = 0, l$, $\mathbf{u_n}$, are the elementary solutions of Saint-Venant [5 - 7],

$$\mathbf{u}(z, \lambda_k) = \mathbf{a}_k e^{i\lambda_k z}$$

$\lambda_k^+ (Im\lambda_k^+ > 0)$, $\lambda_k^- (Im\lambda_k^- < 0)$ eigenvalues of the problem (1.19), $\mathbf{a}_k^+, \mathbf{a}_k^-$ are the corresponding eigenvectors. The SSS responding to these decisions is self-balanced in each cross-section and exponentially damps at a distance from the ends. However, with a weak filler, which in the case under consideration is equivalent to the inequality $E_2/E_1 \ll 1$ (the cross-sectional material is highly inhomogeneous), the "averaging" procedure results in $E \ll E'$, i.e. the "material" obtained after the "averaging" procedure has a strong anisotropy. The studies carried out in [1, 11] for layered plates and cylinders have shown that there exists a finite number $\lambda_k^{\pm}$, the imaginary part of which tends to zero as $m \to 0$. Elementary solutions corresponding to these eigenvalues at the corresponding boundary conditions at the ends, can have a significant effect on the inner cylinder SSS and its rigidity. The presence of weakly damped elementary solutions in the solution of the three-dimensional problem leads to a violation of the Saint-Venant principle for cylindrical bodies of composite materials with a weak filler. In the case of a rope, examples of self-balanced loads can be given which will lead to its destruction, if applied to one of the ends of the rope and the other left free.

2. The problem of tension-torsion

For a cylinder with helical anisotropy, the construction of the Saint-Venant solution of the tensile-contraction problem leads to the following relations [10].

$$2\pi \int_0^a \sigma_{zz} r dr = d_{11}C_1 + d_{12}C_2 = P_z, \;\; 2\pi \int_0^a \sigma_{z\theta} r^2 dr = d_{12}C_1 + d_{22}C_2 = M_z$$

there P_z, M_z are the projections of the principal vector and the principal stress moment acting in the cross section, $C_1 = \varepsilon, C_2 = \varphi$ are arbitrary constants, the first of which can be interpreted as the relative elongation of the rope axis, the second as the relative twist angle of its cross section. The rope will be considered as a composite cylinder described above, for which $E_2 = 0, \nu_2 = 0$. In this case it follows from relations (1.3), (1.4) that which is different from zero only three elements of the matrix of modulus $\mathbf{C}$

$$c_{33} = E' = k_1 E_1, \;\; c_{44} = c_{55} = G' = k_1 G_1$$
$$G' = E'/2(1 + \nu_1)$$

After transforming the corresponding stiffness matrix $\mathbf{C}'$ to the helical coordinate system, we obtain the following expressions for the stresses:

$$\begin{array}{l} \sigma_{rr} = 0, \;\; \sigma_{\theta\theta} = 0, \;\; \sigma_{r\theta} = 0 \\ \sigma_{zz} = c'_{23} e_{\theta\theta} + c'_{33} e_{zz} + c'_{34} e_{\theta z} \\ \sigma_{\theta z} = c'_{24} e_{\theta\theta} + c'_{34} e_{zz} + c'_{44} e_{\theta z} \\ \sigma_{rz} = c'_{55} e_{rz} + c'_{56} e_{r\theta} \end{array} \tag{1.10}$$

$$\begin{array}{l} c'_{23} = \sin(2\beta)^2(-4G' + E')/4, \;\; c'_{33} = \cos(\beta)^4 E' + \sin(2\beta)^2 G', \\ c'_{34} = (\cos(\beta)^2 \sin(2\beta) E' - \sin(4\beta) G')/2, \;\; c'_{44} = \cos(2\beta)^2 G' + \sin(2\beta)^2 E'/4, \\ c'_{55} = \cos(\beta)^2 G', \;\; c'_{56} = \sin(2\beta) G'/2, \quad 0 < \beta < \alpha < \pi/2 \end{array} \tag{1.11}$$

All other elements of the matrix $\mathbf{C}'$ are equal to zero. For the extension problem, the displacement vector has the form

$$\mathbf{u}^{(1)} = (a^{(1)}(r), 0, \varepsilon z + u_z^0) \tag{1.12}$$

u_z^0 is an arbitrary constant, which can be interpreted as a translational displacement parallel to the Oz axis. For a given displacement, different from zero components of the stress tensor have the following form

$$\sigma_{zz}^{(1)} = \frac{E'}{1 + kt^2}\varepsilon, \;\; \sigma_{\theta z}^{(1)} = \frac{E't}{2(1 + kt^2)}\varepsilon$$

For the torsion problem, the displacement vector has the form

$$\mathbf{u}^{(2)} = (a^{(2)}(r), 0, \varphi r z + u_z^0)$$

φ is an arbitrary constant, which can be interpreted as the relative angle of rotation of the cross section of the rope, u_z^0 is an arbitrary constant, which can be interpreted as the angle of rotation of the rope (as a rigid body) around the axis Oz.

For $u^{(2)}$, different from zero components of the stress tensor have the form

$$\sigma_{zz}^{(2)} = \frac{E't}{2(1 + kt^2)} a\varphi, \;\; \sigma_{\theta z}^{(2)} = \frac{E'r}{2(1 + kt^2)}\varphi$$

there

$$t = \beta, \; k = \frac{1 + \nu_1}{2} = \frac{E'}{4G'}$$

The principal vector and principal moment of the stresses acting in the cross section of the rope $z = const$, are determined by the following relations:

$$P_z = d_{11}\varepsilon + d_{12}\varphi, \;\; M_z = d_{21}\varepsilon + d_{22}\varphi$$

where

$$\begin{array}{l} d_{11} = \frac{\pi a^2 E' \ln(1 + kt_1^2)}{t_1 k} \\ d_{12} = d_{21} = \frac{1}{2}\frac{\pi a^3 E'(kt_1^2 - \ln(1 + kt_1^2))}{t_1^2 k^2} \\ d_{22} = \frac{1}{2}\frac{\pi a^4 E'(kt_1^2 - \ln(1 + kt_1^2))}{t_1^3 k^2} \end{array} \tag{1.13}$$

and

$$t_1 = \alpha, \; \alpha = \tau a, \; r = \frac{a\beta}{\alpha}, \; 0 < \beta < \alpha < \pi/2.$$

3. Rigidity of the rope for tension and torsion. It can be seen from relations (1.13) that the rigidity of the rope for tension and torsion depends substantially on the boundary conditions at the ends of the rope $z = 0, L$. Indeed, if the ends are rigidly jammed $(\varphi = 0)$, then it follows from the relations (1.13) that in this case the rigidity to tension

$$D_\varepsilon = d_{11}$$

while in the rope there is a torque $M_z = d_{12}\varepsilon$.

If we assume that the rope is stretched freely suspended by weight P, then in this case, taking into account that $M = 0$, we obtain

$$D_M = d_{11} - d_{12}^2/d_{22}$$

In conclusion, we give graphs illustrating the dependence of D_ε, D_M on the parameter α 10^6 (Fig. 1) [5].

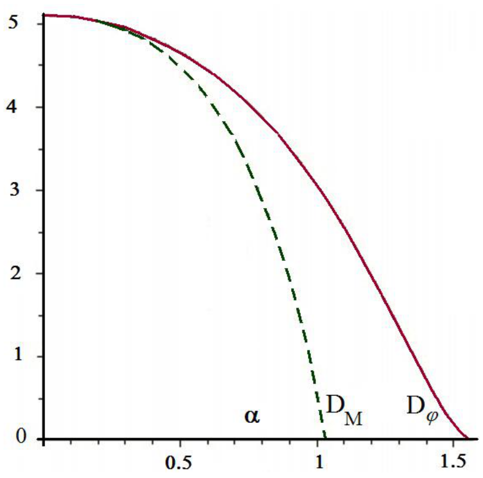

Fig.1. Tension rigidity $D_\varphi(\varphi = 0, M \neq 0)$**,** D_M **(**$M = 0, \varphi \neq 0$**).**

4. Saint-Venant's solutions the bending problems

The rope will be considered as a composite cylinder described above, for which $E_2 = 0, \nu_2 = 0$. In this case, the following relations hold (1.10)-(1.12):

$$c_{33} = E' = k_1 E_1, \ \ c_{44} = c_{55} = G' = k_1 G_1$$

The displacement $\mathbf{C}'$ field of rope points and, accordingly, the strain and stress fields can be described by three groups of elementary solutions [3.8,10].

The first group of six vectors describes the movement of the rope as a rigid body (it is necessary to satisfy the geometric boundary conditions at the ends of the rope). In the helical coordinate system, this group can be represented as:

$$\begin{aligned}
&u_r^0 = C_1 e^{i\psi} + C_2 e^{-i\psi} + zC_3 e^{i\psi} + zC_4 e^{-i\psi}, \\
&u_\theta^0 = iC_1 e^{i\psi} - iC_2 e^{-i\psi} + izC_3 e^{i\psi} - izC_4 e^{-i\psi} + C_6 r, \\
&u_z^0 = -C_3 r e^{i\psi} - C_4 r e^{-i\psi} + C_5, \\
&\psi = \tau z + \theta.
\end{aligned}$$

$$C_1 = \frac{1}{2}(a_1^0 - i a_2^0), \quad C_2 = \overline{C}_1, \quad C_3 = \frac{1}{2}(\omega_2 + i\omega_1), \quad C_4 = \overline{C}_3$$
$$C_5 = a_3^0, \quad C_6 = \omega_3$$

Here a_k^0, ω_k are the projections of the vectors of translational displacement and rotation of the rope as a rigid body on the axis of the basic coordinate system $Ox_1x_2x_3$. A stress-strain state corresponding to the second group is equivalent to the bending moment $M = M_1 + iM_2$ and SSS corresponding to the third group is equivalent to the transverse force $Q = Q_1 + iQ_2$. The elementary solutions that answer them have the form:

$$\mathbf{u_l} = e^{i\psi}\mathbf{a_l}.$$

There

$$\begin{array}{l} \mathbf{a_7} = (z^2/2 + f(r), iz^2/2, rz), \ \ \mathbf{a_8} = \overline{\mathbf{a}}_\mathbf{7} \\ \mathbf{a_9} = (z^3/6 + zf(r), iz^3/6, rz^2/2), \ \ \mathbf{a_{10}} = \overline{\mathbf{a}}_\mathbf{9} \\ f(r) = c'_{23}r^2/4c'_{22} \end{array}$$

For the elementary solution $\mathbf{u_7}$ the corresponding stresses are determined by the following formulas:

$$\begin{array}{l} \sigma_{zz,7} = e^{i\psi}b_{zz,7}, \ \ \sigma_{zz,8} = \overline{\sigma}_{zz,7} \\ \sigma_{z\theta,7} = e^{i\psi}b_{z\theta,7}, \ \ \sigma_{z\theta,8} = \overline{\sigma}_{z\theta,7} \\ \sigma_{rz,7} = e^{i\psi}b_{rz,7}, \ \ \sigma_{rz,8} = \overline{\sigma}_{rz,7} \end{array}$$

There

$$b_{zz,7} = \frac{E't}{(1+kt^2)}, \ \ b_{rz,7} = \frac{G't_1}{2(1+t^2)}, \ \ b_{\theta z,7} = \frac{G'tt_1}{2(1+t^2)}$$

All other components are equal to zero.

For the elementary solution $\mathbf{u_9}$ the stresses are determined by the following formulas:

$$\begin{array}{l} \sigma_{zz,9} = e^{i\psi}(zb_{zz,7} + ib_{zz,9}), \ \ \sigma_{zz,10} = \overline{\sigma}_{zz,9} \\ \sigma_{z\theta,9} = e^{i\psi}(zb_{z\theta,7} + ib_{z\theta,9}), \ \ \sigma_{z\theta,10} = \overline{\sigma}_{z\theta,9} \\ \sigma_{rz,9} = e^{i\psi}(zb_{rz,7} + ib_{rz,9}), \ \ \sigma_{rz,10} = \overline{\sigma}_{rz,9} \end{array}$$

There

$$b_{zz,9} = \frac{E't}{1+t^2}, b_{rz,9} = \frac{G'}{1+t^2}(1+\frac{t_1}{6}), b_{z\theta,9} = \frac{G'}{1+t^2}(1+\frac{tt_1}{6})$$

All other components are equal to zero.

Thus, the vector of displacements and the stress vector of the group elementary solutions can be represented in the form of the following linear combinations:

$$\begin{array}{l} \mathbf{u} = C_7\mathbf{u}_7 + C_8\mathbf{u}_8 + C_9\mathbf{u}_9 + C_{10}\mathbf{u}_{10}, \\ \sigma = C_7\sigma_7 + C_8\sigma_8 + C_9\sigma_9 + C_{10}\sigma_{\mathbf{10}} \end{array}$$

For a rope, as for a cylinder with helical anisotropy, the constants C_k are related to the coordinates of the bending moment M_k and the transverse force Q_k by the following relations [8, 10]:

$$\begin{array}{l} d_{33}C_7 + d_{35}C_9 = M_2 + iM_1 \\ d_{33}C_9 = -Q_1 + iQ_2 \\ C_8 = \overline{C}_7, \ \ C_{10} = \overline{C}_9 \end{array}$$

where Q_j, M_j are the projections of the transverse force and bending moment on the axis of the basic coordinate system $Ox_1x_2x_3$ (Fig. 2),

$$\begin{array}{l} d_{33} = \frac{1}{2}\frac{\pi a^4 kE'}{t_1^2} - \frac{1}{2}\frac{\pi a^4 kE' ln(1+kt_1^2))}{t_1^4 k^2} \\ d_{35} = \frac{4}{3}\pi a^4 G - \frac{4\pi a G}{t_1^2 k^2} + \frac{4\pi a^4 G arctg(kt_1)}{t_1^3 k^3}, \end{array}$$

$$D_{33} = \frac{d_{33}}{\pi a^4 kE}, \ \ D_{35} = \frac{d_{35}}{\pi a^4 G}$$

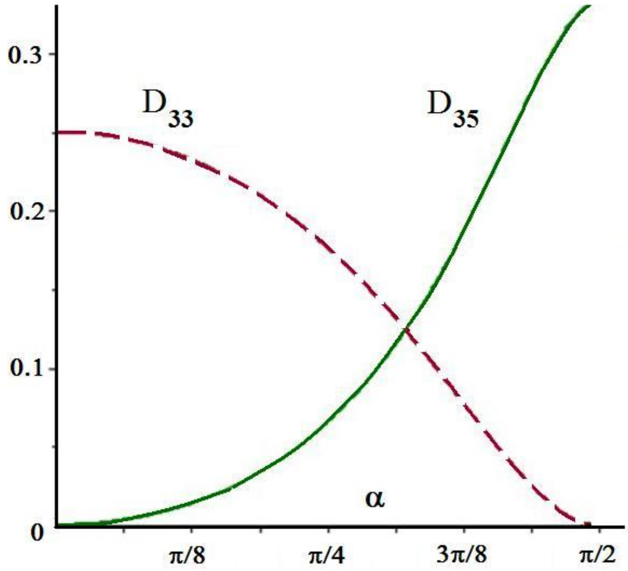

Fig.2. Flexion rigidity $D_{33}D_{35}$.

5. Applied theory of bending ropes

The obtained relationships have a narrow area of application since it is possible to calculate the SSS rope for loads of the bending moment type and transverse force that are applied to its ends. Below we propose another theory, in a sense equivalent to the theory of bending of Bernoulli-Euler rods, which makes it possible to study H under the action of a transverse load on its axis. Let's illustrate this with a concrete example.

Consider the following equation:

$$\begin{array}{l} d_{33}D^4 w = q cos(\theta+\tau z), \quad q = const \\ D = \dfrac{\partial}{\partial z} - \tau\dfrac{\partial}{\partial \theta} \end{array} \qquad (1.14)$$

Here, the right-hand side corresponds to a uniformly distributed transverse load along the axis, directed parallel to the axis Ox_1. Note that for $\theta = 0, \tau = 0$, this equation degenerates into the classical bending equation of the rod under the action of a uniformly distributed load perpendicular to its axis. The particular solution of the inhomogeneous equation in this case has the form

$$w_1 = \frac{qz^4}{24} cos(\theta+\tau z)$$

The general solution of the homogeneous equation can be represented in the form:

$$w_0 = (X_0 + X_1 z + X_2 z^2 + X_3 z^3) cos(\theta+\tau z)$$

there X_k - are arbitrary constants, which are determined by satisfying the boundary conditions at the ends of the rope.

Thus, the general solution of equation (1.12) has the form:

$$w = w_0 + w_1$$

If, for example, the ends of the rope are rigidly embedded, then to determine these constants we obtain the following boundary conditions

$$\begin{array}{l} w(0,\theta) = 0, \;\; \partial_z w(0,\theta) = 0 \\ w(l,\theta) = 0, \;\; \partial_z w(l,\theta) = 0 \end{array}$$

The solution of these equations has the form

$$X_0 = 0, \;\; X_1 = 0, \;\; X_2 = \frac{1}{24}ql^2, \;\; X_3 = -\frac{1}{12}ql \tag{1.15}$$

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