Transactions of NAS of Azerbaijan, Issue Mechanics, **37** (7), 97–100 (2017).
Series of Physical-Technical and Mathematical Sciences.

Determination of vibrations frequencies of a rectangular orthotropic plate with curved structures

Tamella Yu. Zeynalova

Received: 10.09.2017 / Revised: 17.10.2017 / Accepted: 02.11.2017

Abstract. *Following the continual theory, in the paper we study frequency and form of natural vibrations of a rectangular orthotropic plate made of laminated materials with curved structures. The considered composite material consists of alternating layers of two isotropic materials. Solving the problem, for determination frequencies of natural vibrations and eigen functions we get an analytic equation where we can find character of dependence of frequencies of natural vibrations on the parameters of the material and on the curvaturefunction in the structure.*

Keywords. Stress · strain · Hookes law · plate · frequency · vibrations.

Mathematics Subject Classification (2010): 74H45

1 Introduction

The elements of parts and structures made of anisotropic materials are widely used in machine building, construction and in other fields of modern technology. As is known in mechanics of composite materials, the issues related to the features of their structure occupy important place. One on the principal features of the structure of composite materials is the curvature of reinforcing elements [1,2]. Curvature in the structure of a composite material occurs during technological processes as a result of various factors [3]. Therefore it is important to develop the methods to determine the stress strain state and other issues of mechanics of such materials from observations of sections of various composite materials it follows that curvatures in the structure are periodic and local [5].

2 Problem statementin the general form and a method of solution

In the paper we study frequency and form of natural vibrations of a rectangular orthotropic plate made of laminated materials with curved structures. The considered composite material consists of alternating layers of two isotropic materials. It is assumed that the main

T.Yu. Zeynalova
Institute of Mathematics and Mechanics, ANAS
B. Vagabzade 9, Baku, Azerbaijan
E-mail: yoxdur

direction of the plate is parallel to its edges. Mechanical relations of the material are described within the continual theory (1,2). The take the equation of the middle surface of the selected curved layer in the form:

$$x_3 = F(x_1, x_2) = \varepsilon\varphi(x_1 x_2) \tag{2.1}$$

Here ε is a dimensionless small parameter that is determined for every specifically given function of the curvature form $F(x_1 x_2)$ [4,6,7,8]. We assume that for the plate, the equations of the Hooke generalized law with reduced moduli in the matrix form are valid and we write it in the form

$$\left\| \begin{matrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{matrix} \right\| = \left\| \begin{matrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{matrix} \right\| \left\| \begin{matrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{matrix} \right\|, \tag{2.2}$$

where

$$A_{sp} = A_{spo} + \sum_{q=1}^{\infty} \varepsilon^{2q} A_{spq} \left[A_{spo}, \varphi(x_1 x_2) \right] (s : p = 1, 2) \tag{2.3}$$

where A_{spo} are elasticity constants of a homogeneous rectilinear orthotropic plate; A_{spq} are determined by A_{spo} and the parameters of the curvature of the layers [1,2].

Denote by h the thickness of the plate, by u_1 and u_2 displacements of any points in the direction of the axes $x_1$and x_2, by $w(x_1 x_2)$ deflection of the median plane. The form of the function w determines the shape of the curved middle surface. The components of middle surface strains have the form:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = -x_3 \frac{\partial^2 w}{\partial x_1^2}; \quad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = -x_3 \frac{\partial^2 w}{\partial x_2^2};$$

$$\varepsilon_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} = -2x_3 \frac{\partial^2 w}{\partial x_1 \partial x_2}. \tag{2.4}$$

Taking into account the inertia force, the equation of motion will by [2].

$$\frac{\partial^2 M_{11}}{\partial x_1^2} + 2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} + \frac{h\gamma}{g} \frac{\partial^2 w}{\partial t^2} = 0 \tag{2.5}$$

Here M_{11}, M_{22} are bending moments, M_{12}, is twisting moment, γ is specific weight of the plate's material, g is acceleration of gravity.

$$M_{11} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{11} x_3 dx_3, \quad M_{22} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{22} x_3 dx_3, \quad M_{12} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{12} x_3 dx_3 \tag{2.6}$$

Allowing for (2.2) and (2.4), and with regard to curvature of the structure in the direction x_1 the differential vibration equation in the matrix form will be:

$$\frac{h\gamma}{g} \cdot \frac{\partial^2 w}{\partial t^2} + \frac{h^3}{12} \left\{ \left\| A_{11} \;\; 2\left(A_{12} + 2A_{60}\right) \;\; A_{22} \right\| \left\| \begin{matrix} \frac{\partial^4 w}{\partial x_1^4} \\ \frac{\partial^4 w}{\partial x_1^2 \partial x_2^2} \\ \frac{\partial^4 w}{\partial x_2^4} \end{matrix} \right\| + \right.$$

$$\left. +2 \left\| \frac{\partial A_{11}}{\partial x_1} \;\; 2\left(\frac{\partial A_{12}}{\partial x_1} + 2\frac{A_{66}}{\partial x_1} \right) \right\| \left\| \begin{matrix} \frac{\partial^3 w}{\partial x_1^3} \\ \frac{\partial^3 w}{\partial x_1 \partial x_2^2} \end{matrix} \right\| + \left\| \frac{\partial^2 A_{11}}{\partial x_1^2} \;\; \frac{\partial^2 A_{12}}{\partial x_1^2} \right\| \left\| \begin{matrix} \frac{\partial^2 w}{\partial x_1^2} \\ \frac{\partial^2 w}{\partial x_2^2} \end{matrix} \right\| \right\} = 0 \tag{2.7}$$

Flexure w should satisfy boundary conditions dependent on the method of fixing the plate, and initial conditions for

$$t=0 \qquad w=w_0\left(x_1x_2\right) \qquad \frac{\partial w}{\partial t}=v_0\left(x_1x_2\right) \tag{2.8}$$

where $w_0,\ v_0$ is the given initial flexure and initial velocity for the point (x_1x_2). We look for the solution of the equation (2.8) in the form of the product

$$w=\left(C\cos\omega t+D\sin\omega t\right)W\left(x_1x_2\right) \tag{2.9}$$

where ω is the frequency of natural vibrations. Introducing solution of (2.9) in equation (2.7), for W we get the equation

$$\frac{h^3}{12}\left[A_{11}\frac{\partial^4 W}{\partial x_1^4}+2\left(A_{12}+2A_{66}\right)\frac{\partial^4 W}{\partial x_1^2\partial x_2^2}+A_{22}\frac{\partial^4 W}{\partial x_2^4}+2\frac{\partial A_{11}}{\partial x_1}\frac{\partial^3 W}{\partial x_1^3}+\right.$$
$$\left.+2\left(\frac{\partial A_{12}}{\partial x_1}+2\frac{\partial A_{66}}{\partial x_1}\right)\frac{\partial^3 W}{\partial x_1\partial x_2^2}+\frac{\partial^2 A_{11}}{\partial x_1^2}\frac{\partial^2 W}{\partial x_1^2}+\frac{\partial^2 A_{12}}{\partial x_1^2}\frac{\partial^2 W}{\partial x_2^2}\right]-\omega^2 W\frac{h\gamma}{g}=0. \tag{2.10}$$

To equation (2.10) it is necessary to attach the boundary conditions of hinge support

for $x_1=0,\ \ x_1=a \quad W=0, \quad M_{11}=0,$

for $x_2=0,\ \ x_2=b \quad W=0, \quad M_{22}=0.$

We look for the solution of equation (2.10) in the form for:

$$W_{mn}=\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b} \tag{2.11}$$

where $m$and n are integers.

Substituting (2.11) in (2.10), we get

$$\omega^2\frac{h\gamma}{g}\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}=\frac{h^3}{12}\left\{\left[A_{11}\left(\frac{m\pi}{a}\right)^4+2\left(A_{12}+2A_{66}\right)\times\right.\right.$$
$$\left.\times\left(\frac{m\pi^2}{a}\right)\left(\frac{n\pi^2}{b}\right)++A_{22}\left(\frac{n\pi}{b}\right)^2\right]\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}-$$
$$-\left[\frac{\partial^2 A_{11}}{\partial x_1^2}\left(\frac{m\pi}{a}\right)^2+\frac{\partial^2 A_{12}}{\partial x_1^2}\left(\frac{n\pi}{b}\right)^2\right]\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}$$
$$-\left[2\left(\frac{\partial A_{12}}{\partial x_1}+2\frac{\partial A_{66}}{\partial x_1}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2\right.$$
$$\left.\left.+2\frac{\partial A_{11}}{\partial x_1}\left(\frac{m\pi}{a}\right)^3\cos\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}\right]\right\}. \tag{2.12}$$

Accepting

$$A_{sp}=A_{spo}+\varepsilon^2A^2\left(x_1\right)A_{sp1},$$

we use the Bubnov-Galerkin method. Multiply the both hand sides of equality (2.12)

$$\sin\frac{r\pi x_1}{a}\sin\frac{s\pi x_2}{b}$$

and integrate in the entire area of the plate, and taking into account

$$\int\limits_0^a\sin\frac{m\pi x_1}{a}\sin\frac{r\pi x_1}{a}dx_1=\begin{cases}\overline{0} & for \quad m\neq r\\ \frac{a}{2} & for \quad m=r\end{cases}$$

$$\int_0^b \sin\frac{m\pi x_2}{b}\sin\frac{r\pi x_2}{b}dx_2 = \begin{cases}\overline{0} & for \quad n \neq s\\ \frac{b}{2} & for \quad n = s\end{cases}$$

we get

$$\omega = \omega_0 + \frac{\varepsilon\pi^2}{b^2}\sqrt{\frac{g}{h\gamma}\frac{2}{a}}\left[C_1\int_0^a A^2(x_1)\sin^2\frac{m\pi x_1}{a}dx_1 - \right.$$

$$\left.-C_2\int_0^a \frac{\partial^2 A^2(x_1)}{\partial x_1^2}\sin^2\frac{m\pi x_1}{a}dx_1 - C_3\int_0^a \frac{\partial A^2(x_1)}{\partial x_1}\sin 2\frac{m\pi x_1}{a}dx_1\right]^{\frac{1}{2}} \tag{2.13}$$

$$\omega_0 = \frac{\pi^2}{b}\sqrt{\frac{g}{h\gamma}C_0} \tag{2.14}$$

Here $A(x_1) = \frac{\partial\varphi(x_1)}{\partial x_1}$.

$$C_0 = \frac{h^3}{12}\left[A_{110}\left(\frac{m}{c}\right)^4 + 2(A_{120} + 2A_{660})n^2\left(\frac{m}{c}\right)^2 + A_{220}n^4\right]$$

$$C_1 = \frac{h^3}{12}\left[A_{111}\left(\frac{m}{c}\right)^4 + 2(A_{121} + 2A_{661})n^2\left(\frac{m}{c}\right)^2 + A_{221}n^4\right]$$

$$C_2 = \frac{b^2}{\pi^2}\frac{h^3}{12}\left(A_{111}\frac{m^2}{c^2} + A_{121}n^2\right)$$

$$C_3 = \frac{b}{\pi}\frac{h^3}{12}\left[(A_{121} + 2A_{661})\frac{m}{c}n^2 + A_{111}\left(\frac{m}{c}\right)^3\right],$$

ω_{mn}^0 is the frequency of natural vibrations of a straightly laminated orthotropic rectangular plate hingely supported around the entire contour [4].

The expressions for the constants A_{sp0} and A_{sp1} are given in the papers [1, 2, 6, 9]. Solving the problem for determining frequencies of natural vibrations, we found an analytic equation where one can find the character of dependence of the parameter of the material and function of curvature in the structure.

References

1. Akbarov, S.D.: *On stability of a plate Stability Loss and Buckling delamination.* Springer Verlang, Berlin, Heidelberg , 448 (2013).
2. Akbarov, S.D., Goose, A.I.: *On one continual theory of mechanics of composite materials with small-scale curvatures*. Prikladnaya mekhanika, **27** (2), 3–13 (1999).
3. Goose, A.N.: *To the mechanics of composite materials with small-scale curvatures in the structure.* Prikladnaya mekhanika, **XIX** (5), 1–15 (1983)
4. Lekhnitskii, S.G. : Anisotropic plates. *M. Nauka* 635p. (1966).
5. Tarnopolskii, Yu.M., Rose, A.V.: Features of calculation of parts made of reinforced plastics. *Riga, Zinatne*, 279p. (1969).
6. Zeynalova, T.Yu.: *On stability of a rectangular plate made of a laminated composite material*, Nazari va tatbiqi mekhanika. 1-2 Baku, 17.
7. Zeynalova, T.Yu.: *Stability of anisotropic plate with peridically curved structures.* Teoreticheskaya i prikladnaya mekhanika. Baku ELM, (1), 116–120 (2012)
8. Zeynalova, T.Yu.: Stability of anisotropic plate with locally curved structures. *Baku, Mashinsgunaslig,* 2 (2012).
9. Zeynalova, T.Yu.: *Bending of a rectangular anisotropy plate with curved structures*, Proceeding of Institute of Mathematics and Mechanics of NAS of Azerbaijan, **XVIII** (XXVI), 179–186 (2003).