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Strain and elasticity parameters of the medium of the Earth's depth in izotropic approximation

Hatam H. Guliyev · Rashid J. Javanshir · Gular H. Hasanova

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Abstract. *Strain occurs in solids within certain limits continuously. The state of elastic equilibrium is stable within these limits. The change of elastic equilibrium state occurs in reaching strain values of the given limits. The necessity of considering such changes arises in studying of problems on distribution of substance density and other parameters of the Earth's interior structure. These problems are studied in this paper on the basis of nonlinear large (finite) strains within the Lagrangian method. It is shown that strains have concrete intervals of continuous change where conditions of determination of the geological medium parameters are preserved. The boundaries of these intervals are determined by critical values of strains corresponding to various processes of buckling. It is shown that one of sources of faults, inaccuracies and uncertainties is insufficient consideration of features of strains analyzing various experimental and calculation data relating to theoretical models of the Earth.*

Keywords. finite deformation and formation of density of the Earth's substance · instability of deformation process of solid bodies · intervals of continuous change on various criteria of instability

Mathematics Subject Classification (2010): 74A60, 74E15

H.H. Guliyev
Institute of Geology and Geophysics of National Academy of Sciences of Azerbaijan
E-mail: hatamguliyev@gmail.com

R.J. Javanshir
Azerbaijan National Academy of Sciences,
Istiglaliyyat str., 30, Baku, Az1001, Azerbaijan
E-mail: rashidjavanshir51@gmail.com

G.H. Hasanova
Institute of Geology and Geophysics of National Academy of Sciences of Azerbaijan
E-mail: guler.hesenova@yahoo.com

1 Introduction

There are complexes of scientific concepts concerning the formation and development of the geometry of internal constructions and geodynamic-tectonic structures of the Earth. Numerous theoretical models were created using direct data of well (the first 10-metric converter Product ID 12 km 12 km depth of the Earth's crust), geological (sections coming out the daylight surface of the Earth) studies, general (mass, moment of inertia of the Earth, etc.) and indirect data obtained using various geophysical methods. Analysis of results of direct observations, model theoretical and experimental researches show that strain processes play an important role in the formation and development of various structural elements, internal dynamics and the Earth as a whole.

The changes of the equilibrium state, phase transitions and destruction of the medium occur in elastic, elastic-plastic and at subsequent stages of strain. The sequence of their implementation significantly depends on the material content of the geological medium, its stress state, geometry of structures etc. Stress values corresponding to phase transitions (partial melting) are determined in laboratory experiments. Depths are theoretically calculated in the Earth's interior using data on the density of the medium where phase transition can be implemented. Dividing borders were assigned at depths of the Earth comparing the experimental and theoretical results. In addition, the models of internal structure were constructed requiring the implementation of integral conditions of the mechanics concerning the average moment of inertia of the rotating spherical body and total mass of the Earth [3, 10, 11, 12, 23, 38, 41, 40]. Their improvement is continued by adjusting the parameters of models to the data of seismic tomography, deep seismic sounding, natural wobbles of the Earth, geochemistry, petrology, etc. [1, 4, 5, 24, 29, 30, 31, 32]. Correlations were assigned between P, T (P - pressure, T – temperature) conditions and processes of phase transformations for a number of ultramafic rocks in Green, Ringwood, Akimito, Liu and other's famous experimental studies [11, 34, 41]. It follows from these studies that, stress values corresponding to phase transitions are less than stress values corresponding to strength limits of the considered rocks, i.e. process of phase transformations is realized long before the destruction process. Homogeneous samples were tested on composition in experiments. Uniform strains were implemented in samples within uniform compression. Unlike the experimental conditions, the geological medium has various impurities, inclusions at depths of the Earth, disturbances in the structure, etc. Furthermore, inhomogeneity of chemical composition (mineral associations) of rocks and ascending streams (mainly gas) of interstitial elements contribute the geological medium becomes at least a two-phase system with various correlations of crystalline and amorphous phases [13]. Therefore, deformation processes of rocks significantly differ in natural conditions and samples in laboratory experiments. This difference becomes even more significant due to uniform deformation within various homogeneous and inhomogeneous stress states.

The advanced technologies are used in modern experiments [2, 35] to obtain thermobaric conditions corresponding to deeper bowels of the Earth. According to them, the experiments are conducted using the dynamic method in quite short time intervals. Therefore, there is no way to track the lengthy formation processes in general. The concept on fast kinetics used in the method of wave pressure is based on shear strain of the matter. But these strain are caused during the experiment to register the results of measurement and don't apply to strain taking place in natural evolutionary process of the Earth. More visual concept on strain processes is outlined through the analogy with the mechanics of strain of fibrous and layered composites naturally. It is known that [6, 15] small and large curvatures of the reinforcing fibers and layers occur in their structure as a result of deformation of the composite materials. As a result, there are significant changes in the distribution of strain and stress in the medium. In its turn, these changes are reflected in mechanical-strength properties of the material. Such researches refer to a priority direction in the theory of composite materials and develop intensively [5, 6, 7, 15].

Interesting results were given [28]. Anomalies in velocity and rheological characteristics of some basalts and silicates were observed in laboratory experiments at high pressure. It is considered that one of the main reasons of these anomalies is local changes in structures in compression process. Experimental results of crystallization of the molten material are obtained under the conditions of similar conditions of solid inner core [35]. These and similar numerous experimental data are basic in modeling the inner core as solid body consisting of iron alloys.

Similar but more complex, diverse and extensive deformation processes are implemented in the layered composite structures of the Earth that can't only be described by uniform strains [17, 18]. In principle, the correct consideration the role of strains processes in the structural and geodynamic-tectonic evolution of the Earth will also allow going further in the understanding of the problem ("the Dynamics of the Earth") [1].

It's necessary to make the following remark. Various criteria are suggested in the mechanics of standard materials and constructions on the basis of the theory of strength, limit states, plasticity, mechanics of destruction of the fractured bodies, durability and the theory of stability to determine allowable limit of strains. As a usual, these strains don't exceed 1-2% at elastic stage. The exceptions make rubber-like materials that can undergo large strains. The problem on strains values is different in geology. The materials are deformed for a long geological time in the Earth's interior and strains values may be different. The achievement of strains values in local zones of any limit values determined within the above mentioned theories don't mean that the loss of strength occurs or capability of the composite construction of the Earth is exhausted as a whole. These limit values indicates possible processes in local zones in this case. Depending on the structural, geometric features of these zones, physical-mechanical properties of materials, forms and values of impacts in the Earth's interior, different strains processes such as the curvature of the layered structures, the formation of faults and cavities, the expansion of the existed faults, plastic flow, delamination, and loosening and consolidation of the media, partial melting, etc. can be implemented. As a result, significant redistribution of stress and deformations occur in the considered zones. Such changes can create the term of implementation of the fault and formation of shear and layered zones even at extremely high normal pressure (at a depth of 700 km and more) as mechanical processes (it is noted in [13] based on the concept of uniform strains of homogeneous medium that the indicated mechanical processes cannot be implemented at such depths). Moreover, it is not necessary that these processes should be directly related to processes of interaction of the lower fields of the mantle with the outer liquid core. Seismotomographic researches show that extensive deconsolidated regions and diverse channels of mass flow are far from the borders of the outer core in the Earth's interior [37].

Summarizing the above mentioned, it should be noted that the geological medium acts as a material on the one hand and as an element of construction on the other hand in the Earth's interior. Strains nature of homogeneous, composite material and elements of construction differ significantly. It is necessary to properly consider this fact in theoretical models of development of the Earth and in the interpretation of results of experimental studies.

Great success in geophysics, physics of high pressure and temperature, seismotomography, deep seismic sounding, wobbles of the Earth, tidal strains and nutations, surface waves, geomechanical and petrological studies gave a lot of information on the internal structure and the occurred processes. An enormous array of data was accumulated on various parameters of the internal structure of the Earth and this array continues to fill by flows of new data on various directions. The parameters of the established theoretical models are clarified by accumulation of new data. A key component of all models is the solution of problems of density distribution and determining the state law of matter on depth [4, 11, 22, 23, 40, 41].

There are faults and uncertainties in the obtained results for objective and subjective reasons. Faults and errors in the final results are in the form of inaccuracies in experimental and observational data, approximations of theoretical basis of processing and interpretation and uncertainties in our scientific concepts. In principle, there is no opportunity to fully evaluate the accuracy reliability of observational and experimental (assigning of which is primarily based on our subjective assumption) of data. In addition, the consistency of data aggregate of various researches (geological, geophysical, physical-chemical, mechanical, etc.) and their compliance with the experimental (invented by us) results and current scientific concepts are required. This is the essence of the idea on the reliability of observational (direct and indirect) data. There is an impression on the need of development of systematic approach for processing and interpretation of it, always updating array of various nature and degree of reliability. At the same time, it is necessary to develop an approach based on the minimum number of observational and experimental data covering the assumed mechanisms of flow of processes more completely. In this regard, numerous studies were implemented [1, 4, 5, 10, 24, 29, 30, 31, 32, 38, 41]. The areas of allowable values of certain mechanical parameters of the lithosphere, mantle, outer liquid and inner solid core were determined in them.

The main idea of studying the problems of distribution of various fundamental parameters of theoretical models of the Earth's development consists of their consideration at present. The history of their formation is excluded. Despite the fact that such an approach significantly reduces the number of variants, uncertainty and diversity remains in solving this fundamental problem of the Earth's science.

In case of uniform compression, distribution problem in the density of the Earth's material on depth was studied [9] using Eulerian method within the theory of finite strains. As a result, the law of Birch-Murnaghan's state was offered [9, 10]. Further, it was shown in [27] that significantly different result is obtained in case of applying Lagrangian method to describe the strains process. Various authors discussed the given problem subsequently. The bibliography of these studies is provided [4]. Theoretical results of both approaches were applied for interpretation of various observational and experimental data and the supposed mechanisms of formation of structures within the model representations. Limits of applicability of various state equations were determined. Experimentally determined restrictions imposed on physico-mechanical properties of materials (e.g. the moduli of shear and pressure must not be negative, etc.) were used for these purposes. The consistency of calculation data with model representations were considered a measure of reliability results. The disadvantages of such approaches are mainly related with two reasons. First, theoretical dependencies of various methods are used to describe strain processes within one and the same model. Second, many details on mineralogical compositions, physical-mechanical, petrological, geochemical and other properties of the material are required. Uncertainty is high in the degree of homogeneity of the mineral composition of rocks. It is considered that values of various observational and experimental data must be within the intervals of stable change of fundamental physical-mechanical and chemical parameters [4]. Such approaches are time-consuming and related with great risks. At the same time, the probability of involvement of significant errors and inaccuracies are quite high in the results.

Dependencies between the results of Lagrangian and Eulerian method had already been obtained for a long time in the mechanics of the deformed solids to describe strain processes. There are simple analytical correlations between the main values of Green's strain tensor (Lagrangian method) and Almansi (Eulerian method) [36, 15]. Regarding the convenience in the use and procedures of mathematical simplication, methods of description of nonlinear strain processes have their advantages and disadvantages in various problems. For an instance, it is preferable to use Lagrangian method in studying of problems of stability of equilibrium state. In this case, the position and form of the deformable body is considered to be known in natural state that allows simplifying the solution of quite difficult mathematical problem. From this point of view, it is also convenient to apply Lagrangian method in the problem of distribution of the density of the Earth's material as the material density is considered to be known in the unstrained state. Moreover, in case of experimental researches of density, the medium density is known (directly measured) behind the front of pressure wave and in front of it using shock waves of pressure.

There is no method allowing determining the countless number of parameters of deformable system (impact components and impact objects) in more or quite less complete volume in practice within real terms (i.e. in various unreachable depths of the Earth's interior considering the geological time). Despite the fact that, quite acceptable results describing certain aspects of the given problem was obtained in some concrete cases, in general, such situation has led to the creation of numerous models of the Earth and obtaining of ambiguous and uncertain conclusions [3, 4, 11, 12, 23, 40].

Finally, it is shown that the solution of problem of the Earth's interior structures and distribution of elastic parameters in them is not only sufficient on the basis of integral criteria [20, 21]. Sufficient local conditions follow from the requirements of the mechanics of continuous medium, and it is necessary to achieve their implementation in solving structural problems on distribution of elastic parameters

It is suggested to base on problems of the nonlinear theory of mechanics of continuum media assigned in strains in this paper with the purposes of achieving reliable and ambiguous (certain) results on density distribution of material and other parameters of the Earth's model within current geodynamics. Nonlinear theory of deformation of solids is applied as a theoretical apparatus within Lagrangian method to describe them and nonlinear linearized approach considering small and large (finite) strains.

The strains considers all-possible forms and natures of impact, the degree of the medium exposure to impacts, physical-mechanical, petrophysical, geochemical, thermal and other characteristics of the deformable system in a complete scale. At the same time, in case of considering the problems in strains, there is no need in concrete data on the above mentioned properties as well as in state equations. The strain is the most universal parameter in the system, object and results of impacts. It's necessary to evaluate this parameter in the deformable system and to use it in theoretical, experimental researches and interpretations properly. Consequently, first of all, it is necessary to define fields of stable continuous change (in case of pressure increase) of strain and then to determine other necessary parameters and conduct the interpretation of different data within it. Critical values of strains determining the intervals of keeping the given term is achieved within different processes, and their implementation mechanisms can be caused by impacts of various nature.

2 The medium density within the finite strain

The following equation was obtained in nonlinear theory of deformable solids using general theoretical dependence of determination of geometric objects (length, fields and volume) and the law of the mass conservation [15]

$$\frac{\rho}{\rho_0} = I_3^{-\frac{1}{2}};$$
$$I_3 = 1 + 2A_1 + 2(A_1^2 - A_2) + \frac{4}{3}(2A_3 - 3A_2A_1 + A_1^3); \tag{2.1}$$
$$A_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3; A_2 = \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2; A_3 = \varepsilon_1^3 + \varepsilon_2^3 + \varepsilon_3^3.$$

Here ε_i $(i = 1, 2, 3)$ are the main components of Green's strain tensor in describing strains by Lagrangian method; ρ is medium density in the current state; ρ_0 is medium density in natural (undeformed) state.

Formulae (2.1) in [19] were used to study the density of the Earth's materials considering inhomogeneities of uniform strains.

It follows from the structure (2.1) that it is necessary to know the numerical value of its initial densityρ_0 and the main components of Green's deformation tensor ε_i in the current state. Knowledge on types of stress states, their nature, state equations, parameters of state equations, etc. are not required in using this universal correlation.

In case of homogeneous strains we can assume without loss of generality that

$$\varepsilon_1 = \alpha\varepsilon_0, \varepsilon_2 = \beta\varepsilon_0, \varepsilon_3 = \gamma\varepsilon_0, \tag{2.2}$$

where ε_0 is the parameter of uniform strain, α, β, γ are material numbers. It is possible to consider different non-uniform homogeneous strain state estimating these numbers correctly. Considering (2.2) in (2.1), we obtain a more convenient formula for calculations

$$\frac{\rho}{\rho_0} = \left(1 + A\varepsilon_0 + B\varepsilon_0^2 + C\varepsilon_0^3\right)^{-\frac{1}{2}},$$

$$A = 2\left(\alpha + \beta + \gamma\right), B = 4\left(\alpha\beta + \alpha\gamma + \beta\gamma\right), C = 8\alpha\beta\gamma. \tag{2.3}$$

Formulae (2.1) - (2.3) cover the stages of small and large strains.

In case of uniform strain ($\alpha = \beta = \gamma = 1$) formula (2.3) is simplified and takes quite simple form

$$\frac{\rho}{\rho_0} = \left(1 + 2\varepsilon_0\right)^{-\frac{3}{2}}. \tag{2.4}$$

Formula (2.4) takes the following form for an isotropic elastic body within compression

$$\frac{\rho}{\rho_0} = \left(1 - x\right)^{-\frac{3}{2}}; x = \frac{2\left(1 - 2\nu\right)}{E}P, \tag{2.5}$$

where P is pressure per unit of the area in the unstrained state (in case of small elastic strains); in case of large strains, pressure can also be correlated to the unit of the area of the initial strained state; ν is Poisson's ratio; E is Young's modulus of elasticity.

It follows from the formulae (2.1) - (2.4) that assigning the medium density on any stage of strain (i.e. at atmospheric pressure) its value is determined at the subsequent changes of strains without use of data on state equation and on the system of external impacts. In contrast, it's seen from the formula (2.5) that there is an additional necessity in data on physical-mechanical properties of the medium (Poisson's ratioν and elasticity modulus$E$) in assigning stress (pressure).

Graphs of change in density depending on change of non-uniform strains were given in Fig.1.

The following equation was obtained by Euler's method [9, 11]

$$\frac{\rho}{\rho_0} = \left(1 - 2\varepsilon_0\right)^{\frac{3}{2}}. \tag{2.6}$$

The equation of Birch-Murnaghan's state was offered in case of finite uniform deformations based on the formula (2.6)

$$\frac{2}{3K_0^{'}}P = \left(\frac{\rho}{\rho_0}\right)^{\frac{7}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{5}{3}}. \tag{2.7}$$

Similarly, state equation was obtained using formula (2.4) [27]

$$\frac{2}{3K_0^{'}}P = \left(\frac{\rho}{\rho_0}\right)^{-\frac{1}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{3}}. \tag{2.8}$$

K_0' is isothermal pressure modulus in these formulae.

It follows from these results that the description of deformation process by Lagrange and Euler's method gives significantly different results in the considered problems. This state was the subject of discussion for a long time and finally, it was solved in [4], i.e. it was indicated that the main values of Almansi and Green's different strain tensors were used in these formulae accordingly. According to the above mentioned, there is a simple formula of transition among these values. Formulae (2.4) and (2.6) are equivalents. However, they must be applied accurately in theoretical models of the Earth, i.e. all theoretical dependencies should be described either within Lagrangian or Eulerian method. The problems on determination of areas of applicability of various tectonophysical parameters, as well as correlation (2.4) is solved below evaluating the limits of change of strains of uniform compression from the point of view of three-dimensional theory of elastic stability of equilibrium states.

3 Confidence interval on strains

3.1. The "Internal" instability

The problems on stability of strain process are one of intensively studied fields of the mechanics of strained solids [8, 7, 14, 15, 16, 17].

The elastic equilibrium state of the strained isotropic body (within the compressed model of the medium)

$$\lambda_1^* < \lambda_1 < 1 \tag{3.1}$$

is stable [16]. Limit values of elongation (shortening) coefficient determining the interval of stability assign concrete structures (forms) of elastic potentials. In case of modeling of strain process using harmonic elastic potential

$$\lambda_1^* = \frac{1+\nu}{2-\nu}, \tag{3.2}$$

λ_1 is elongation along the coordinate axes; ν is Poisson's ratio.

It is known that $0 < \nu < 0.5$. Consequently, we obtain the value $0.5 < \lambda_1^* < 1$ for ultimate elongation λ_1^* for all possible materials. In case of homogeneous initial finite strain [16]

$$\lambda_1^2 = 1 + 2\varepsilon_0. \tag{3.3}$$

we determine the field of its change using formulae (3.2) and (3.3) for the deformation parameter

$$\varepsilon_0^* < \varepsilon_0 < 0; \varepsilon_0^* = \frac{3}{4}\frac{2\nu-1}{(2-\nu)^2}, \tag{3.4}$$

where the equilibrium state is stable.

In case of modeling, strain process is determined in the following form using quadratic elastic potential λ_1^*

$$\lambda_1^* = \left(\frac{1+\nu}{2-\nu}\right)^{\frac{1}{2}}. \tag{3.5}$$

Using formulae (3.3) and (3.5), we determine the critical parameter of strain

$$\varepsilon_0^* = \frac{1}{2}\frac{2\nu-1}{2-\nu}. \tag{3.6}$$

The field of stability is also determined as the inequality (3.4) considering (3.6) in this case.

We obtain the following equations on the basis of these results to determine possible intervals of density change of the Earth's material in cases of harmonic and quadratic elastic potentials of finite strains respectively

$$1 \leq \frac{\rho}{\rho_0} \leq \left(\frac{\rho}{\rho_0}\right)^*; \left(\frac{\rho}{\rho_0}\right)^* = (1 + 2\varepsilon_0^*)^{-\frac{3}{2}} = \left(\frac{2-\nu}{1+\nu}\right)^3, \tag{3.7}$$

$$1 \leq \frac{\rho}{\rho_0} \leq \left(\frac{\rho}{\rho_0}\right)^*; \left(\frac{\rho}{\rho_0}\right)^* = \left(\frac{2-\nu}{1+\nu}\right)^{\frac{3}{2}}. \tag{3.8}$$

The achievement of shortening and strains of their critical values (3.2), (3.5) and (3.4), (3.6) corresponds to "internal" instability [16]. All points of the unlimited homogeneous isotropic medium (within the compressed model) get the finite disturbance in achieving of critical values. These values of strains determine the theoretical limit of durability of the considered materials [16]. It is considered that the body not undergone any geometric forming is destructed in all points simultaneously. Consequently in this case, the formulae (2.1), (2.3)-(2.5) correctly describe the change in the medium density up to the valueε_0^*. Consolidation process of the medium is changed into deconsolidation process in achieving this critical value of strain.

Thus, formulae (2.4), (2.7) interpreting experimental and observation data must be applied to describe density distribution and state equation of the Earth's material keeping the term imposed on deformation $\varepsilon_0^* < \varepsilon_0 < 0$. In case of applying harmonic and quadratic elastic potentials, limits of these inequalities are characterized using formulae (3.4) and (3.6)-(3.8). It is also possible to derive the corresponding correlation for other forms of elastic potentials of finite strains similar to formulae (3.4) and (3.6)-(3.8).

3.2. Instability of strain on geometric forming

The inequations mentioned above allow theoretically determining the possible limits of variation of the studied parameters within the accepted conditions. These intervals may be different in actual practice due to the properties of strain process even at the elastic stage. If there are inhomogeneities of physical-mechanical and geometric origin in the medium, more complex strain processes can occur in it. Nonuniform distribution of strains can be caused by the process of changing of elastic equilibrium within the uniform pressure in certain situations, i.e. due to the loss of stability.

Let's consider the case of uniform strain of the medium influenced by conservative ("dead") external loads. The state of elastic equilibrium body loses the stability at lower values $(\varepsilon_0)_*$than critical strains ε_0^* at such impact. Let's consider the stability of elastic equilibrium state of half-space in the vicinity of a vertical cylindrical cavity of circular cross-section as a specific example. The "dead" loads were defined on the cylindrical surface of the cavity, intensity of which is equal to the value of intensity of the external load acting on the "infinity" along horizontal planes. In this case, state of homogeneous deformation is implemented as follows:

$$u_m^0 = \delta_{im} (\lambda_i - 1) x_i,$$

where $x_i \equiv x^i$ $(i = 1, 2, 3)$ are Lagrangian coordinates which coincide with the Cartesian coordinates; δ_{im} Kronecker's symbols; u_m^0 constituents of displacement vector in the initial state; $\lambda_i$elongation along coordinate axis; direction of coordinate lines coincide with main directions of strain tensor.

Three-dimensional nonlinear problem of stability was studied in such formulation in [26]. Unfortunately, unlike the case of "tracer" loads, it is not possible to make conclusions about the stability and instability in the general form irrespective of the elastic potentials and body shape in the considered variant. Therefore, the problems were separately studied using different elastic potentials. It was shown that the state of elastic equilibrium is unstable in the vicinity of the cylindrical cavity in case of influencing on its surface by conservative forces . The following analytical expressions were obtained for critical values of elongation corresponding to the loss of stability of the equilibrium state with geometric forming:

in case of harmonic potential

$$(\lambda_1)_* = -\frac{(1+\nu)(1+2\nu)}{2(2+\nu-4\nu^2)}\left\{1-\left[1+\frac{8(2+\nu-4\nu^2)}{(1+2\nu)^2}\right]^{\frac{1}{2}}\right\}; \tag{3.9}$$

in case of quadratic potential

$$(\lambda_1)_* = \left(\frac{3}{3-2x}\right)^{\frac{1}{2}}; x = -\frac{3(5-4\nu)}{16(1+\nu)}\left\{1-\left[1-\frac{16(1-2\nu)}{(5-4\nu)^2}\right]^{\frac{1}{2}}\right\}. \tag{3.10}$$

The corresponding values of critical strains $(\varepsilon_0)_*$ are determined using the formula (3.3) considering (3.9) and (3.10). The body loses the stability and gets the geometric shape through the changing of the existed geometric shape with more stable state of equilibrium in achieving the values of deformations of critical values $(\varepsilon_0)_*$. The distribution of stress and deformations becomes nonuniform in a new state. Therefore, the values of change of the medium density determined using formulae (2.1), (2.3)-(2.5) will be equal to critical values of strains $(\varepsilon_0)_*$ in each of its point. The values of the change of the medium density will vary after the loss of stability in various points of the body.

Numerical results are shown in Table 1 for critical values of shortening and strains at various data of Poisson's ratio calculated on formulae (3.2), (3.4)-(3.6), (3.9) and (3.10). The results corresponding to harmonic were given in the numerator, and quadratic in the denominations. It follows from these results that the loss of stability of elastic equilibrium state caused by "internal" instability occurs in the vicinity of the cylindrical cavity in this case. Stable intervals of change of strains and density instead of inequations (3.4), (3.5) and (3.7), (3.8) are determined from the following inequations under such circumstances:

$$(\varepsilon_0)_* < \varepsilon_0 < 0; \varepsilon_0^* < (\varepsilon_0)_*; \tag{3.11}$$

$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)_*; \left(\frac{\rho}{\rho_0}\right)_* \le \left(\frac{\rho}{\rho_0}\right)^*; \left(\frac{\rho}{\rho_0}\right)_* =$$

$$= [1+2(\varepsilon_0)_*]^{-\frac{3}{2}}; (\varepsilon_0)_* = \frac{1}{2}\left[(\lambda_1)_*^2 - 1\right]. \tag{3.12}$$

Thus, changing of equilibrium state is implemented by loss of stability in case of influence of conservative forces on the cylindrical surface in bodies described by harmonic and quadratic elastic potentials. The body undergoing geometric forming in the local vicinity of the cavity gets more stable curved geometric shape before the beginning of destruction process. In its turn, this forming leads to unequal distribution of stress and strains in the vicinity of the cavity. This conclusion is not only related with solving the considered specific problem but also carries a general character. The fact is that according to three-dimensional theory of stability of the deformable solid bodies, the loss of stability of equilibrium state (local in case of model of unrestricted bodies; general forms in cases of restricted bodies) with geometric form change is necessary in their vicinity with increase in strain values in the presence of lines or conservative forces in strained system. In all such cases, initial terms of homogeneous distribution of strain in the body are broken and distribution of strain becomes non-homogeneous.

Before discussing the numerical results, it should be noted that the obtained results can be generalized in this section in case of elastic-plastic stage of strain. In case of elastic-plastic deformation, nonclassical linearization provides new possibilities to acquire approximate solutions. Making a supposition, those zones of a discharge may only appear in the initial-strained state and it is possible to solve difficult nonlinear problems considering processes of active subsidence being in a perturbed state. Such approach is called a summarized conception of a continuous subsidence in the theory of elastic-plastic strain [15].

4 Numerical results and discussions

In Table 1, along with the above mentioned discussion, numerical values of critical values of density corresponding to the "internal" instability and loss of stability of equilibrium state changes on geometric form. They show that the loss of stability of equilibrium state on geometric forming leads to "internal" instability, i.e. to the beginning of destruction process in the considered case for all values of Poisson's ratio. Similar processes of local loss of stability of the equilibrium state occur in the vicinity of the existing inclusions in the form of rods, bands, plates, etc. Therefore, the use of inequations (3.11)-(3.12) is more reasonable and correct in practice, especially in researches of the Earth's crust and upper mantle. When the structural inhomogeneity is included to the medium by the cylindrical rod or band from more rigid media, it is known [16, 26], that straight form of rod becomes instable and gets more stable curved shape in uniform compression in quite small strains by conservative forces. In this case, the critical strength of stability loss is two times less than Eulerian's force corresponding to uniaxial compression along the rod line. This force is many times less than the pressure of partial melting and phase transitions for different substances of the Earth. Depending on geometric dimensions of the inclusion, their curvatures may lead to significant local changes (dissimilarity from uniform strain) of strains and stresses in a large scale.

Such local strain processes will influence on further change of density and other tectonophysical parameters in large geometric scales and for a geological time. In particular, processes of partial melting and phase transitions will not occur at single deep levels of the Earth's interior as in uniform strain. In real conditions, these processes will be implemented at different levels of the Earth's interior depending on the nature of strain distribution.

It's seen from the structure of formulae (3.2), (3.4)-(3.10) that, it's necessary to know only values of Poisson's ratio to determine critical values of elongation (shortening) and strains. This fact is of great practical importance. There are different geophysical methods allowing determining this important physico-mechanical parameters of the medium within conditions of unreachable deep interior of the Earth. It is possible to conduct assessment of various theoretical and observation results and to determine their degree of reliability using these data and inequations proposed in this paper.

Hofmeister [22] analyzed the questions of applicability of theoretical results to determine certainty of fields of Birch-Murnaghan's state equation (B-M EoS) received within Euler's tasking of finite strain. Strains were modeled using equivalent repulsive interatomic potentials. As the criteria of applicability (certainty) of results, the term of saving the interval of change of isothermal modulus of compression was applied within the framework of which the potential structure remains stable. Based on [39] experimental results, there was concluded that, theoretical model B-M EoS is unreliable for some solids as orthopyroxene. Let's consider this problem from the point of view of the above received inequalities.

It was assigned in the known Lin-Gun Liu's [34, 41] experiments that as a result of uniform strain of sample from orthopyroxene $(90\% MgSiO_3 \cdot Al_2O_3)$, phase transition from enstatite into garnet occurs in a relative decrease of the volume by $7,8\%$. The decrease in the volume by $8,0\%$ causes a new phase transition from garnet into ilmenite. The further decrease in the volume by $6,9\%$ leads to the phase transition from ilmenite into perovskite. Using the formula (2.4) and these experimental data, numerical calculations were conducted and their results were reflected in the Fig.2.

Results show that enstatite undergoing uniform strain of pressure at its 2,64%, 5,34% and 7,69% values is sequently changed into garnet, ilmenite and at last, into perovskite. An increase in the density of matter by 8,45%, 8,7% and 7,4% corresponds to these values of strains. Comparison of these results with data in Table 1 regarding ε_0^*, show that the sequence of phase transitions of orthopyroxene causes the destruction in all values of Poisson's ratio as a result of continuous strain (in its modeling by harmonic potential). A similar conclusion is obtained for a quadratic potential except of the interval of changing the values of Poisson's ratio $\nu > 0.38$. Comparison of the results on the parameter $(\varepsilon_0)_*$ show that the local loss of stability of the elastic equilibrium state can cause separate phase transitions in the vicinity of inclusions in the form of the cylindrical cavity for a range of changes of Poisson's ratio $\nu > 0.38$ (harmonic potential) and $\nu \geq 0.12$ (quadratic potential). In such situation, the obtained results cannot be considered reliable due to violation the term of uniformity of strain process. It is known [33], that the value of Poisson's ratio averaged in Voigt-Reuss-Hill's approximation changes within $0,19 \leq \nu \leq 0,21$ in the interval of temperature $25^0C \leq T \leq 700^0C$ for orthopyroxene. Thus, results obtained within uniform strains must be adjusted in case of presence the inhomogeneity as inclusion the form of the cylindrical cavity in orthopyroxene medium. The given conclusion relates to the case when strain process is modeled by quadratic elastic potential. As a result of local instability, equal character of distribution of pressure strain is broken in the medium. Therefore, nature of density distribution and other tectonophysical parameters will differ from analogical nature corresponding to the case of uniform strain in case of uniform compression. Furthermore, it follows from results in the Fig. 2 that the parameters of physico-mechanical properties of orthopyroxene are undergone significant changes in the strained state due to the realized phase transitions. Numerical values of parameters of physical-mechanical properties of these rocks differ among themselves significantly. This example shows quite clearly the difficulties in assessing the confidence intervals of state equations on criteria of fundamental moduli of the elasticity. Apparently, the conclusions on the unreliability of the model B-M EoS are related with the mentioned circumstances here to describe experimental data of orthopyroxene.

The results given in Fig.1 show that one and the same density changes can occur in a variety of combinations between the main components of Green's strain tensor differing from uniform compression.

Thus, even in uniform strains, failure of compression uniformity, implementation on various mechanisms of instability of elastic equilibrium state (instability of equilibrium state can also be implemented at stages of elastic-plastic, elastic-viscous and other stages of strains) and phase transitions may have a significant impact on distribution of the medium density, depth of implementation of phase transitions and other parameters of theoretical models of the Earth. Hence, determination of material and other parameters of theoretical models can lead to inaccurate conclusions only on the basis of results of experimental researches on separate physical-mechanical characteristics and phase transitions in mineral associations under uniform strain.

Considering the above mentioned, it should be noted that, imposed ones on strains are "strong". The parameters of theoretical models of the Earth must be determined within the proposed intervals of change of strains. In other approaches based on terms of saving restrictions imposed on separate parameters of state equations (depending on, how successive are equations themselves), correspondences of their results with the indicated interval of strain aren't tested and true mechanisms of strain in the considered questions are left out. Consequently, "weak" restrictions on separate parameters including to various correlation of theoretical models of development of the Earth were formulated within such approaches.

It means that the required term could be implemented on separate parameters but they will not provide unambiguity of interpretation and uncertainties in results.

Thus, proper involvement the theory of finite strains to problems of determining different parameters of theoretical models of the Earth's development allow formulating reasonable strain criteria on their confidence intervals. Apparently, these criteria are the most universal, simple and comfortable to apply.

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Table 1. Critical values ε_0 and $\frac{\rho}{\rho_0}$

ν	0	0.1	0.2	0.3	0.4	0.5
$(\lambda_1)_*$	$\frac{0.78}{0.89}$	$\frac{0.82}{0.91}$	$\frac{0.84}{0.94}$	$\frac{0.89}{0.95}$	$\frac{0.94}{0.98}$	$\frac{1}{1}$
λ_1^*	$\frac{0.50}{0.71}$	$\frac{0.58}{0.76}$	$\frac{0.67}{0.82}$	$\frac{0.77}{0.87}$	$\frac{0.87}{0.94}$	$\frac{1}{1}$
$(\varepsilon_0)_*$	$\frac{-0.1958}{-0.1040}$	$\frac{-0.1638}{-0.0860}$	$\frac{-0.1472}{-0.0582}$	$\frac{-0.1040}{-0.0488}$	$\frac{-0.0582}{-0.0198}$	$\frac{0}{0}$
ε_0^*	$\frac{-0.375}{-0.2480}$	$\frac{-0.3318}{-0.2112}$	$\frac{-0.2756}{-0.1638}$	$\frac{-0.2036}{-0.1216}$	$\frac{-0.1216}{-0.0582}$	$\frac{0}{0}$
$\left(\frac{\rho}{\rho_0}\right)_*$	$\frac{2.1073}{1.4185}$	$\frac{1.8137}{1.3270}$	$\frac{1.6872}{1.2040}$	$\frac{1.4185}{1.1664}$	$\frac{1.2040}{1.0625}$	$\frac{1}{1}$
$\left(\frac{\rho}{\rho_0}\right)^*$	$\frac{8}{2.7940}$	$\frac{5.1253}{2.2780}$	$\frac{3.3249}{1.8137}$	$\frac{2.1904}{1.5186}$	$\frac{1.5186}{1.2040}$	$\frac{1}{1}$

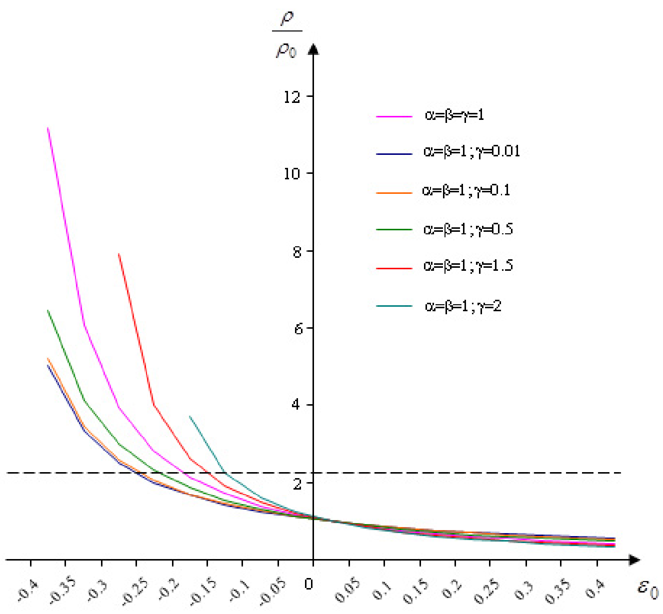

Fig. 1. $\frac{\rho}{\rho_0}$ **dependence on** ε_0**.**

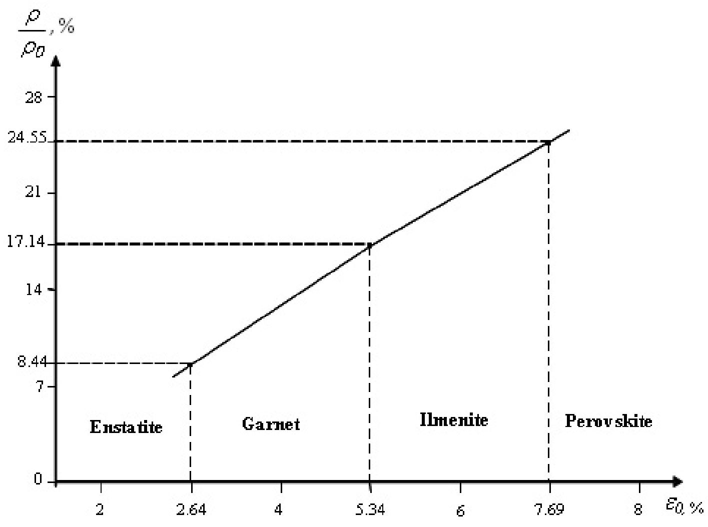

Fig. 2. $\frac{\rho}{\rho_0}$ **dependence on** ε_0 **and sequence of phase transitions for orthopyroxene.**

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