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Analysis of the second boundary value problem of elasticity theory for a small thickness inhomogeneous transwersally-isotropic cone

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Abstract. *The second boundary-value problem of the theory of elasticity for inhomogeneous transversely isotropic cone of small thickness have been studied using the method of asymptotic integration of the equations of the theory of elasticity. Non-uniform solutions are constructed. The nature of the homogeneous solutions constructed is studied. It is shown that for an inhomogeneous transversally isotropic cone with a fixed lateral surface, the stress-strain state is made up only from a solution having the nature of a boundary layer.*

Keywords. inhomogeneous solutions · homogeneous solutions · boundary layer · transversally isotropic cone · Saint-Venant's edge effect.

Mathematics Subject Classification (2010): 74D05

1 Introduction

The papers [1, 2] concentrates on the asymptotic method axially symmetric problems of elasticity theory for a variable thickness conical shell. In the papers [3, 4] the general theory of a transversally isotropic conic shell is developed. The axisymmetric problems of the theory of elasticity for a conic shell were investigated on the basis of the asymptotic method in this paper.

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2 Problem statement

We consider an axially symmetric problem of elasticity theory for an inhomogeneous transversal cone that is an isotropic body with two conic and two spherical boundaries. Lets classify the cone as a spherical coordinate system r, θ, φ and denote the domain occupied by the cone by $\Gamma = \{r \in [r_1; r_2], \theta \in [\theta_1; \theta_2]; \varphi \in [0; 2\pi]\}$. Equilibrium equations at the absence of mass fores in spherical system of coordinates has the form [7]:

$$\begin{cases} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{2\sigma_{rr} - \sigma_{\varphi\varphi} - \sigma_{\theta\theta} + \sigma_{r\theta} ctg\theta}{r} = 0, \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{3\sigma_{r\theta} + (\sigma_{\theta\theta} - \sigma_{\varphi\varphi}) \cdot ctg\theta}{r} = 0 \end{cases} \quad (2.1)$$

where σ_{rr},$\sigma_{r\theta}$, $\sigma_{\varphi\varphi}$, $\sigma_{r\varphi}$, $\sigma_{\varphi\theta}$, are stress tensor components expressed by the displacement vector components $u_r = u_r(r, \theta)$, $u_\theta = u_\theta(r, \theta)$ in the following way [8]:

$$\sigma_{rr} = a_{11}\frac{\partial u_r}{\partial r} + a_{12}\left(\frac{u_\theta}{r} ctg\theta + 2\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right),$$

$$\sigma_{\varphi\varphi} = a_{12}\frac{\partial u_r}{\partial r} + (a_{22} + a_{23})\frac{u_r}{r} + a_{22}\frac{u_\theta}{r} ctg\theta + a_{23} \cdot \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \quad (2.2)$$

$$\sigma_{\theta\theta} = a_{12}\frac{\partial u_r}{\partial r} + (a_{22} + a_{23})\frac{u_r}{r} + a_{23}\frac{u_\theta}{r} ctg\theta + a_{22} \cdot \frac{1}{r}\frac{\partial u_\theta}{\partial \theta},$$

$$\sigma_{r\theta} = a_{44}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial \theta} - \frac{u_\theta}{r}\right).$$

Substituting (2.2) in (2.1) we get an equilibrium equations in displacements

$$\left(L_0 + \varepsilon\partial_1 L_1 + \varepsilon^2 \partial_1^2 L_2\right)\overline{w} = \overline{0} \quad (2.3)$$

where L_k are matrix differential operators of the form

$$L_0 \left\| \begin{matrix} \partial(b_{44}\partial) + 2\varepsilon^2(b_{12} - b_{22} - b_{23}) + \\ +\varepsilon b_{44} ctg(\theta_0 + \varepsilon\eta)\partial \end{matrix} \quad \begin{matrix} \varepsilon(b_{12} - b_{22} - b_{23})\partial + \varepsilon^2(b_{12} - b_{23} - \\ -b_{44} - b_{22}) ctg(\theta_0 + \varepsilon\eta) - \varepsilon\partial(b_{44}) \end{matrix} \\ \begin{matrix} \varepsilon\partial((b_{22} + b_{23})) + 2\varepsilon b_{44}\partial \end{matrix} \quad \begin{matrix} \partial(b_{22}\partial) + \varepsilon\partial(b_{23} ctg(\theta_0 + \varepsilon\eta)) - 2\varepsilon^2 b_{44} + \\ +(b_{22} - b_{23}) \cdot \varepsilon ctg(\theta_0 + \varepsilon\eta)\partial - \\ -(b_{22} - b_{23})\varepsilon^2 ctg(\theta_0 + \varepsilon\eta) \end{matrix} \right\|,$$

$$L_1 = \left\| \begin{matrix} 2\varepsilon b_{11} & \partial(b_{44}) + b_{12}\partial + \varepsilon(b_{44} + b_{12}) ctg(\theta_0 + \varepsilon\eta) \\ \partial(b_{12}) + b_{44}\partial & 2\varepsilon b_{44} \end{matrix} \right\|$$

$$L_2 = \left\| \begin{matrix} b_{11} & 0 \\ 0 & b_{44} \end{matrix} \right\| \quad \partial = \frac{\partial}{\partial \eta}, \ \partial_1 = \rho\frac{\partial}{\partial \rho}, \ \partial_1^2 = \rho^2\frac{\partial^2}{\partial \rho^2},$$

$$\overline{W} = (W_\rho, W_\theta)^T, \ W_\rho = W_\rho(\rho; \eta), \ W_\theta = W_\theta(\rho; \eta).$$

There $\eta = \frac{\theta - \theta_0}{\varepsilon}$, $\rho = \frac{r}{r_2}$ are new dimensionless variables; $\varepsilon = \frac{\theta_2 - \theta_1}{2}$ - characterized the cone's thickness; $\theta_0 = \frac{\theta_2 + \theta_1}{2}$- is the opening angle of the median surface of the cone; $\eta \in [-1; 1]$; $\rho \in [\rho_1; 1]$; $\theta_0 \in \left(0; \frac{\pi}{2}\right)$; $W_\rho = \frac{u_r}{r_2}$; $W_\theta = \frac{u_\theta}{r_2}$, $b_{11} = \frac{a_{11}}{G_0}$; $b_{12} = \frac{a_{12}}{G_0}$, $b_{22} = \frac{a_{12}}{G_0}$, $b_{23} = \frac{a_{23}}{G_0}$, $b_{44} = \frac{a_{44}}{G_0}$ - are dimensionless variables; G_0 -as some characteristic parameter having the size of elasticity modulus a_{ij}.

It is assumed that elasticity module $b_{ij} = b_{ij}(\eta)$ are arbitrary positive conditions function of variable η whose values may change about one order

Let on the lateral surfaces of the cone the following boundary conditions are given

$$\overline{W}\big|_{\eta=\pm 1} = \overline{g}(\rho). \tag{2.4}$$

Here $\overline{g}^{\pm}(\rho) = (f^{\pm}(\rho), h^{\pm}(\rho))^T$.

We assume that $f^{\pm}(\rho)$, $h^{\pm}(\rho)$ are sufficiently smooth functions and relatively ε hand the order $O(1)$.

3 Construction of inhomogeneous solutions

Let us construct inhomogeneous solutions, i.e. the solutions of equation (2.3) satisfying boundary conditions (2.4).

We look for the solution of (2.3), (2.4) in the form:

$$\begin{array}{l} W_\rho = W_{\rho 0} + \varepsilon W_{\rho 1} + \cdots \\ W_\theta = W_{\theta 0} + \varepsilon W_{\theta 1} + \cdots \end{array} \tag{3.1}$$

Substitution of (3.1) in (2.3), (2.4) reduces to the system whose successive integration gives the relations for the expansion coefficients of (3.1):

$$u_{\rho 0} = \frac{f(\rho)}{\tau_1}\psi_1(\eta) + f^-(\rho), \tag{3.2}$$

$$u_{\theta 0} = \frac{h(\rho)}{\tau_2}\psi_2(\eta) + h^-(\rho). \tag{3.3}$$

$$\begin{gathered} u_{\rho 1} = \frac{h(\rho)}{\tau_2}\left(\int_{-1}^{\eta}\frac{\psi_3(x)}{b_{44}(x)}dx + \int_{-1}^{\eta}\psi_2(x)dx\right) - \frac{\rho h'(\rho)}{\tau_2}\int_{-1}^{\eta}\frac{\psi_4(x)}{b_{44}(x)}dx - \\ -ctg\theta_0\frac{f(\rho)}{\tau_1}\left(\psi_1(\eta)+\psi_5(\eta)\right) + (\eta+1)h^-(\rho) - \\ -\frac{\rho h'(\rho)}{\tau_2}\int_{-1}^{\eta}\psi_2(x)dx - (\eta+1)\rho(h^-(\rho))' + C_1(\rho)\psi_1(\eta); \end{gathered} \tag{3.4}$$

$$\begin{gathered} u_{\theta 1} = -\frac{(2f(\rho)+\rho f'(\rho))}{\tau_1}(\psi_2(\eta)+\psi_6(\eta)) + \frac{h(\rho)ctg\theta_0}{\tau_2}\times \\ \times\left(\int_{-1}^{\eta}\frac{\psi_7(x)}{b_{22}}dx - \psi_6(\eta) - \psi_2(\eta) - \int_{-1}^{\eta}\frac{b_{23}}{b_{22}}\psi_2(x)dx\right) - \\ -ctg\theta_0\cdot h^-(\rho)\psi_7(\eta) - \rho(f^-(\rho))'\psi_4(\eta) - f^-(\rho)(\psi_7(\eta)+\eta+1) - \frac{\rho f'(\rho)}{\psi_1(\eta)}\times \\ \times\int_{-1}^{\eta}\frac{b_{12}(x)}{b_{22}(x)}\psi_1(x)dx - \frac{f(\rho)}{\varphi_1}\left(\int_{-1}^{\eta}\psi_1(x)dx + \int_{-1}^{\eta}\frac{b_{23}(x)}{b_{22}(x)}\psi_1(x)dx\right) + C_2(\rho)\psi_2(\eta); \end{gathered} \tag{3.5}$$

$$C_1(\rho) = \frac{1}{\tau_1}\left[ctg\theta_0\frac{f(\rho)}{\tau_1}\left(\tau_1+\psi_5(1)\right) - \frac{h(\rho)}{\tau_2}\times\right.$$

$$\left(\tau_2-\psi_6(1)+\int\limits_{-1}^{1}\frac{\psi_3(\eta)}{b_{44}(\eta)}d\eta\right)-2h^-(\rho)+\frac{\rho h'(\rho)}{\tau_2}\times$$

$$\times\left(\tau_2-\psi_6(1)+\int\limits_{-1}^{1}\frac{\psi_4(\eta)}{b_{44}(\eta)}d\eta\right)+2\rho\left(h^-(\rho)\right)'\Big]; \qquad (3.6)$$

$$C_2(\rho)=\frac{1}{\tau_2}\left[\frac{(2f(\rho)+\rho f'(\rho))\cdot(\tau_2+\psi_6(1))}{\tau_1}+\frac{h(\rho)ctg\theta_0}{\tau_2}\times\right.$$

$$\times\left(\psi_6(1)+\tau_2+\int\limits_{-1}^{1}\frac{b_{23}}{b_{22}}\psi_2(\eta)d\eta-\int\limits_{-1}^{1}\frac{\psi_7(\eta)}{b_{22}}d\eta\right)+ctg\theta_0\cdot h^-(\rho)\psi_7(\eta)+$$

$$+\rho(f^-(\rho))'\cdot\psi_4(\eta)+(2+\psi_7(1))\cdot f^-(\rho)+\frac{\rho f'(\rho)}{\tau_1}\int\limits_{-1}^{1}\frac{b_{12}}{b_{22}}\psi_1(\eta)d\eta+$$

$$\left.+\frac{f(\rho)}{\tau_1}\left(\tau_1-\psi_5(1)+\int\limits_{-1}^{1}\frac{b_{23}}{b_{22}}\psi_1(\eta)d\eta\right)\right]; \qquad (3.7)$$

where

$$\tau_1=\int\limits_{-1}^{1}\frac{1}{b_{44}}d\eta;\quad \tau_2=\int\limits_{-1}^{1}\frac{1}{b_{22}}d\eta;$$

$$\psi_1(\eta)=\int\limits_{-1}^{\eta}\frac{1}{b_{44}}dx;\ \psi_2(\eta)=\int\limits_{-1}^{\eta}\frac{1}{b_{22}}dx$$

$$f(\rho)=f^+(\rho)-f^-(\rho);\quad h(\rho)=h^+(\rho)-h^-(\rho);$$

$$\psi_3(\eta)=\int\limits_{-1}^{\eta}\frac{b_{22}+b_{23}-b_{12}}{b_{22}}dx,\ \ \psi_4(\eta)=\int\limits_{-1}^{\eta}\frac{b_{12}}{b_{22}}dx,$$

$$\psi_5(\eta)=\int\limits_{-1}^{\eta}\frac{x}{b_{44}}dx,\ \ \psi_6(\eta)=\int\limits_{-1}^{\eta}\frac{x}{b_{22}}dx,\ \ \psi_7(\eta)=\int\limits_{-1}^{\eta}\frac{b_{23}}{b_{22}}dx.$$

The stress with respect to ε have the order $O(\varepsilon^{-1})$.

4 Construction and analysis of homogeneous solutions

Assume in (2.4) $\overline{g}^{\pm}(\rho) = \overline{0}$:

$$\overline{W}\big|_{\eta=\pm 1} = \overline{0}. \tag{4.1}$$

Any solution of equation (2.3) satisfying the conditions (4.1) is said to be a homogeneous solution.

Looking for the solution of (2.3), (4.1) in the form

$$\overline{W}(\rho,\eta) = \rho^{z-\frac{1}{2}}\overline{u}(\eta) \tag{4.2}$$

we get the spectral problem

$$\begin{cases} \left(L_0 + \varepsilon\left(z-\frac{1}{2}\right)(L_1 - \varepsilon L_2) + \varepsilon^2\left(z-\frac{1}{2}\right)^2 L_2\right)\overline{u}(\eta) = \overline{0} \\ \overline{u}(\eta) = \overline{0} \quad \text{as} \quad \eta = \pm 1 \end{cases} \tag{4.3}$$

where

$$\overline{u}(\eta) = (a(\eta), c(\eta))^T$$

As $\varepsilon \to 0$ for the solution (4.3) we apply the asymptotic method based on there iterative processes [6]. The homogeneous solutions of (4.3), corresponding to the first asymptotic process can be obtained from (3.1) (3.7), if we substitute in the equation $\overline{g}^{\pm}(\rho) = \overline{0}$. In this case we get trivial solutions correspond to the first asymptotic process.

There is no solution that has the edge effect characterize and corresponds to the first asymptotic process for a cone with fastened lateral surface.

According to the asymptotic process we look of for the solution of (4.3) in the form:

$$\begin{aligned} a^{(3)}(\eta) &= \varepsilon(a_{30} + \varepsilon a_{31} + \ldots.), \\ c^{3}(\eta) &= \varepsilon(c_{30} + \varepsilon c_{31} + \ldots.), \\ z &= \varepsilon^{-1}(\beta_0 + \varepsilon\beta_1 + \ldots) \end{aligned} \tag{4.4}$$

After substitution (4.4) in (4.3) for the first expansion terms, we get the spectral problem

$$L(\beta)\overline{u}_0 = \left\{ l_0(\beta_0)\overline{u}_0; \ \ \overline{u}_0|_{\eta=\pm 1} = \overline{0} \right\} = \overline{0} \tag{4.5}$$

where

$$l_0(\beta_0) = L_{00} + \beta_0 L_{10} + \beta_0^2 L_2; \ \ \overline{u}_0 = (a_{30}; c_{30})^T$$

$$L_{00} = \left\| \begin{matrix} \partial(b_{44}\partial) & 0 \\ 0 & \partial(b_{22}\partial) \end{matrix} \right\|,$$

$$L_{10} = \left\| \begin{matrix} 0 & \partial(b_{44}) + b_{12}\partial \\ \partial(b_{12}) + b_{44}\partial & 0 \end{matrix} \right\|$$

(4.5) describes potential solution of plate transversely isotropic inhomogeneous in thickness's [5, 9, 10].

At the following stage, for determining $\overline{u_1} = (a_{31}, c_{31})^T$ β_1 we get the boundary value problem:

$$\begin{cases} (L_{00} + \beta_0 L_{10} + \beta_0^2 L_2)\overline{u}_1 = -\beta_1(L_{10} + 2\beta_0 L_2)\overline{u}_0 - \beta_0 L_{11}\overline{u}_0 - L_4\overline{u}_0, \\ \overline{u_1}|_{\eta=\pm 1} = \overline{0} \end{cases} \tag{4.6}$$

where

$$L_{11} = \left\| \begin{matrix} 0 & (b_{44} + b_{12})ctg\theta_0 \\ 0 & 0 \end{matrix} \right\|,$$

$$L_4 = \left\| \begin{array}{cc} b_{44} ctg\theta_0 \partial & (\frac{b_{12}}{2} - b_{22} - b_{23})\partial - \frac{3}{2}\partial(b_{44}) \\ \partial(b_{22} - b_{23} - \frac{b_{12}}{2}) - \frac{3}{2} b_{44}\partial & \partial(b_{23}) ctg\theta_0 + (b_{22} - b_{23}) ctg\theta_0 \partial \end{array} \right\|$$

The solubility condition of (4.6) is orthogonality of the right hand side of the solution of the associated problem

$$L^*(\beta_0)\overline{u}_0^* = L(-\overline{\beta}_0)\overline{u}_0^* = \overline{0}$$

where $\overline{u}_0^* = (a_{30}^*; c_{30}^*)^T$.

Satisfying the solubility conditions for β_1 we have:

$$\beta_1 = \frac{D_0}{D_1},$$

where

$$D_1 = \int_{-1}^{1} \left[b_{12} a_0 \frac{d\overline{c}_0^*}{d\eta} - b_{44} \frac{da_0}{d\eta} \cdot \overline{c}_0^* - b_{12} \overline{a}_0^* \frac{dc_0}{d\eta} + b_{44} c_0 \frac{d\overline{a}_0^*}{d\eta} - \right.$$

$$\left. - 2\alpha_0 \left(b_{44} c_0 \overline{c}_0^* + b_{11} a_0 \overline{a}_0^* \right) \right] d\eta;$$

$$D_0 = \int_{-1}^{1} \left[\frac{3}{2} b_{44} \overline{c}_0^* \frac{da_0}{d\eta} + (b_{22} - b_{23}) \frac{dc_0}{d\eta} \overline{c}_0^* ctg\theta_0 - \left(b_{22} + b_{23} - \frac{b_{12}}{2} \right) \times \right.$$

$$\times \frac{dc_0}{d\eta} \overline{a}_0^* + \alpha_0 (b_{44} + b_{12}) c_0 \overline{a}_0^* ctg\theta_0 + b_{44} \frac{da_0}{d\eta} \overline{a}_0^* ctg\theta_0 - b_{23} c_0 \frac{d\overline{c}_0^*}{d\eta} ctg\theta_0 -$$

$$\left. - \left(b_{22} + b_{23} - \frac{b_{12}}{2} \right) a_0 \frac{d\overline{c}_0^*}{d\eta} - \frac{3}{2} b_{44} c_0 \cdot \frac{d\overline{a}_0^*}{d\eta} \right] d\eta.$$

By means of the substitution

$$a_{30}(\eta) = \beta_0^{-2} g_0 f''(\eta) - g_1 f(\eta),$$

$$c_{30}(\eta) = -\beta_0^{-3} (g_0 f''(\eta))' + \beta_0^{-1} (g_2 f'(\eta) + \beta_0^{-1} (g_1 f(\eta))'$$

the problem (4.5) is reduced to the Popkovich generalized spectral problem [5, 9, 10]:

$$\begin{cases} (g_0 f''(\eta))'' - \beta_0^2 \left[(g_1 f(\eta))'' + g_1 f''(\eta) + (g_2 f'(\eta)') \right] + \beta_0^4 g_3 f(\eta) = 0, \\ g_0 f''(\eta) - \beta_0^2 g_1 f(\eta) = 0 \quad \text{as} \quad \eta = \pm 1. \\ (g_0 f''(\eta))' - \beta_0^2 \left[g_2 f'(\eta) + (g_1 f(\eta))' \right] = 0 \quad \text{as} \quad \eta = \pm 1. \end{cases} \tag{4.7}$$

where

$$g_0 = \frac{b_{22}}{b_{12}^2 - b_{11} b_{22}}; \quad g_1 = \frac{b_{12}}{b_{12}^2 - b_{11} b_{22}}; \; g_2 = \frac{1}{b_{44}}; \quad g_3 = \frac{b_{11}}{b_{12}^2 - b_{11} b_{22}}$$

The solution corresponding to the third integrating process have the form:

$$u_\rho^{(3)}(\rho; \eta) = \varepsilon \rho^{-\frac{1}{2}} \sum_{k=1}^{\infty} D_k \left[\beta_{0k}^{-2} g_0 f_k''(\eta) - g_1 f_k(\eta) + \right.$$

$$\left. + O(\varepsilon) \right] \cdot \exp\left(\left(\frac{\beta_{0k}}{\varepsilon} + \beta_{1k} \right) \cdot \ln \rho \right), \tag{4.8}$$

$$u_\theta^{(3)}(\rho; \eta) = \varepsilon \rho^{-\frac{1}{2}} \sum_{k=1}^{\infty} D_k \left[-\beta_{0k}^{-3} (g_0 f_k''(\eta))' + \beta_{0k}^{-1} g_2 f_k'(\eta) + \right.$$

$$+\beta_{0k}^{-1}(g_1 f_k(\eta))' + O(\varepsilon)] \exp\left(\left(\frac{\beta_{0k}}{\varepsilon} + \beta_{1k}\right)\cdot \ln\rho\right).$$

Solutions of (4.8) have the boundary lager character and the first terms of (4.8) equivalent to Saint-Venant's edge effect of inhomogeneous transversally-isotropic plate [5, 9].

Assume that on the spherical part of the boundary of the shell the boundary conditions are given

$$\sigma_{\rho\rho} = f_{1S}(\eta), \quad \sigma_{\rho\theta} = f_{2S}(\eta) \quad \rho = \rho_s \ \ (s = 1; 2)$$

where $f_{1s}(\eta), f_{2s}(\eta)$ are sufficiently smooth functions satisfying the equilibrium conditions.

For determining the constants D_k based on Lagrange's variational principle, we get the infinite system of linear algebraic equations:

$$\sum_{k=1}^{\infty} g_{kn} D_{n0} = h_k; \ \ (k = 1, 2, ...,). \tag{4.9}$$

where

$$D_k = D_{k0} + \varepsilon D_{k1} +, ...,$$

$$g_{kn} = \int_{-1}^{1} \left\{-\beta_{0n}^{-1} f_n'' \left[\beta_{0k}^{-2} g_0 f_k'' - g_1 f_k\right] + f_n' \left[\beta_{0k}^{-1}(g_1 f_k)' + \beta_{0k}^{-1} g_2 f_k' - \beta_{0n}^{-3}(g_1 f_k'')'\right]\right\} d\eta \times$$

$$\times \sum_{s=1}^{2} \exp\left(\left(\beta_{1n} + \beta_{1k} + \frac{(\beta_{0n} + \beta_{0k})}{\varepsilon}\right) \ln\rho_s\right),$$

$$h_k = \sum_{s=1}^{2} \rho_s^{3/2} \exp\left(\left(\frac{\beta_{0k}}{\varepsilon} + \beta_{1k}\right)\cdot \ln\rho_s\right) \cdot \int_{-1}^{1} \left\{f_{1s}(\eta)\left[\beta_{0k}^{-2} g_0 f_k'' - g_1 f_k\right] + \right.$$

$$\left. + f_{2s}(\eta)\left[-\beta_{0k}^{-3}(g_0 f_k'')' + \beta_{0k}^{-1} g_2 f_k' + \beta_{0k}^{-1}(g_1 f_k)'\right]\right\} d\eta$$

Definition of D_{kj} $(j = 1, 2, ...)$ invariably reduces to inversion of one and the same matrices that coincide with the matrices of system (4.10)

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