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Free fluctuations of the of the plates suspend in the various ways

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Abstract. *On the field of the plate dynamics it is considered the oscillation and waves in different cases. In the plate cases there is no obvious expression of the frequency. In this work considered the tasks of the vibration of the plats by the different cases lean on and the case of a hinge with elastic lean, round mass with an elastic lean and soon. It is calculates the lean stiffness as the function of the frequency.*

Keywords. oscillations · frequency · radius · stiffness · plates.

Mathematics Subject Classification (2010): 74K20

1 Introduction

Circular plates are widely applied in various branches of technics as working elements in aircraft engineering, in civil building, etc.

To definition of own frequenciesof fluctuations of circular plates as free and based upon the elastic basis of type of Winkler, a number of works [2], [6], [1] is devoted. Only for static problems about a bend of the rectangular plates lying on the elastic basis with variable factor of bed, decisions are known. In article [7] calculation of such plates is conducted by a method of final elements, and in [3] - a method of Galyorkin.

In the given work free fluctuations of a round plate are considered at various kinds of a suspension bracket. It will be obvious to affect a suspension bracket kind and frequency of fluctuations. In practice plate fastening can appear distinct from planned and consequently the knowledge is necessary as fastening influences frequency of fluctuations. In work [5] symmetric cross-section fluctuations metallic-polymer the three-layer circular plate connected with the elastic basis are investigated, at a heatstroke. For external layers hypotheses of Kirhgof are accepted, in an easy filler the deformed normal is rectilinear and

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no compressible on a thickness. Analytical decisions are received,their numerical analysis is carried out.

2 Case momentless plates (membrane).

The equation of movement looks like:

$$\frac{\partial^2 W}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial W}{\partial r} = \frac{1}{a^2} \frac{\partial^2 W}{\partial t^2}, \tag{2.1}$$

where W- moving.
At an oscillative motion the equation (2.1) with the account $W = W_0 \sin \omega t$ becomes:

$$\frac{\partial^2 W_0}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial W_0}{\partial r} + \frac{\omega}{a^2} W_0 = 0. \tag{2.2}$$

For elastic fastening lowering an index W_0 $r = r_0$(fig. 1)

$$\frac{\partial W}{\partial r} = qW, \tag{2.3}$$

where q- a fastening constant.

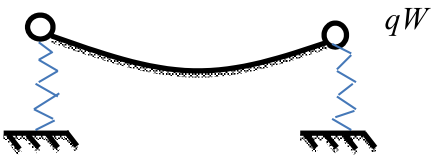

Fig.1.

The decision of the equation (2.2)

$$W = CJ_0\left(\frac{\omega r}{a}\right). \tag{2.4}$$

Substituting (2.4) in (2.3) it is had

$$-\frac{\omega}{a} \cdot J_1\left(\frac{\omega r_0}{a}\right) = qJ_0\left(\frac{\omega r_0}{a}\right). \tag{2.5}$$

Having presented (2.5) in a kind

$$q = -\frac{\omega}{a} \frac{J_1\left(\frac{\omega r_0}{a}\right)}{J_0\left(\frac{\omega r_0}{a}\right)}. \tag{2.6}$$

We have q as function from frequency ω or a spectrum $q \to \omega$. For $a = 1400\frac{M}{sec}, r = 10M$ the spectrum $q \to \omega$ on Fig. 2 is resulted.

Further problems of free fluctuations of round plates are considered at various variants of a suspension bracket.

The equation of plate fluctuations looks like [4]:

$$\varDelta\varDelta W - \beta^4 W = 0, \tag{2.7}$$

where $\beta^2 = \frac{r_0^4 q\omega^2}{D}$; $D = \frac{Eh^3}{12(1-\mu)^2}$- cylindrical rigidity of a plate, q- the weight of a plate carried to unit of a surface, r_0- plate radius, h- a thickness, ω- frequency of fluctuations, μ- factor of Poisson, Δ- operator Laplace.

The decision of the equation (2.1) looks like:

$$W = AJ_0(\beta\rho) + BI_0(\beta\rho), \tag{2.8}$$

where $\rho = \frac{r}{a}$.
Further derivatives $J_0\left(\frac{\omega r_0}{a}\right)$and $I_0\left(\frac{\omega r_0}{a}\right)$ functions of Bessel will be necessary.

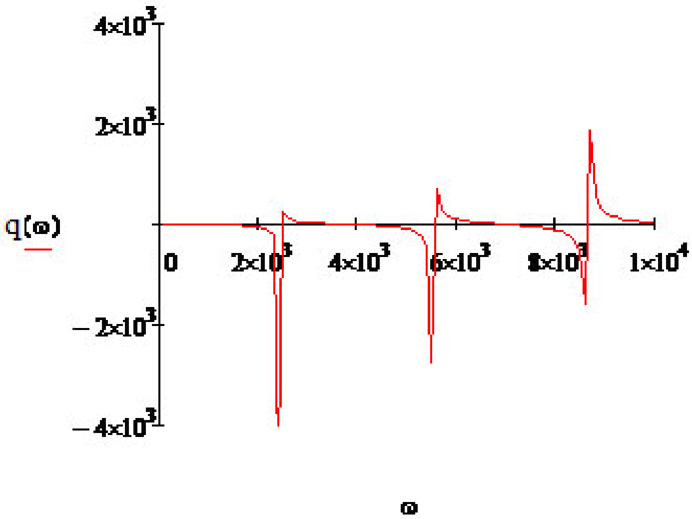

Fig.2.

3 The plate is hinge fixed on a contour and is elastic leans (Fig. 3)

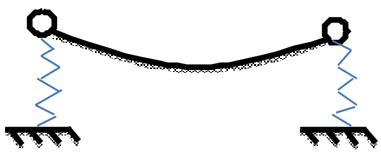

Fig.3.

The condition hinged looks like fastening:

$$\frac{d^2W}{dr^2} + \frac{\nu}{r} \cdot \frac{dW}{dr} = 0. \tag{3.1}$$

Condition elastic supporting at $r = r_0$

$$\frac{d^3W}{dr^3} + \frac{\nu}{r}\frac{d^2W}{dr^2} - \frac{\nu}{r^2}\frac{dW}{dr} = \eta W. \tag{3.2}$$

Substituting (2.8) in (3.1) and (3.2) it is received, considering

$$\frac{\partial J_0\left(\frac{\omega r}{a}\right)}{\partial r}=-\frac{\omega}{a}J_1\left(\frac{\omega r}{a}\right)\quad;\frac{\partial I_0\left(\frac{\omega r}{a}\right)}{\partial r}=-\frac{\omega}{a}I_1\left(\frac{\omega r}{a}\right)$$

$$\frac{\partial^2 J_0\left(\frac{\omega r}{a}\right)}{\partial r^2}=\frac{\omega^2}{a^2}\left[\frac{a}{\omega r}J_1\left(\frac{\omega r}{a}\right)-J_0\left(\frac{\omega r}{a}\right)\right]\quad;$$

$$\frac{\partial^2 I_0\left(\frac{\omega r}{a}\right)}{\partial r^2}=\frac{\omega^2}{a^2}\left[\frac{a}{\omega r}I_1\left(\frac{\omega r}{a}\right)-I_0\left(\frac{\omega r}{a}\right)\right]$$

$$\frac{\partial^3 J_0\left(\frac{\omega r}{a}\right)}{\partial r^3}=\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)J_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}J_0\left(\frac{\omega r}{a}\right)\quad;$$

$$\frac{\partial^3 I_0\left(\frac{\omega r}{a}\right)}{\partial r^3}=\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)I_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}I_0\left(\frac{\omega r}{a}\right)\quad;$$

$$A\frac{\omega^2}{a^2}\left[\frac{a}{\omega r}J_1\left(\frac{\omega r}{a}\right)-J_0\left(\frac{\omega r}{a}\right)\right].$$

$$B\frac{\omega^2}{a^2}\left[\frac{a}{\omega r}I_1\left(\frac{\omega r}{a}\right)-I_0\left(\frac{\omega r}{a}\right)\right]-\frac{\nu}{r}\frac{\omega}{a}AJ_1\left(\frac{\omega r}{a}\right)-\frac{\nu}{r}\frac{\omega}{a}BI_1\left(\frac{\omega r}{a}\right)=0. \tag{3.3}$$

$$A\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)J_1\left(\frac{\omega r}{a}\right)+B\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)$$
$$\times I_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}J_0\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}I_0\left(\frac{\omega r}{a}\right)$$
$$+\frac{\nu}{r}\frac{\omega^2}{a^2}\left[A\left(\frac{a}{\omega r}J_1\left(\frac{\omega r}{a}\right)-J_0\left(\frac{\omega r}{a}\right)\right)+B\left(\frac{a}{\omega r}I_1\left(\frac{\omega r}{a}\right)-I_0\left(\frac{\omega r}{a}\right)\right)\right]$$
$$-\frac{\nu}{r^2}\frac{\omega}{a}\left(AJ_0\left(\frac{\omega r}{a}\right)+BI_0\left(\frac{\omega r}{a}\right)\right)=\eta\left(AJ_0\left(\frac{\omega r}{a}\right)+BI_0\left(\frac{\omega r}{a}\right)\right). \tag{3.4}$$

Let's receive from (3.3)

$$A=-\frac{B\cdot\left(\left(\frac{\nu}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)I_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2}I_0\left(\frac{\omega r}{a}\right)\right)}{\left(\frac{\nu}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)J_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2}J_0\left(\frac{\omega r}{a}\right)}. \tag{3.5}$$

Substituting (3.5) in (3.4) it is had

$$\eta(\omega)=\left(\left(\frac{v}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)I_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2}I_0\left(\frac{\omega r}{a}\right)\right)$$
$$\times\frac{\left(\left(\frac{\omega^3}{a^3}-\frac{2}{r^2}\cdot\frac{\omega}{a}+\frac{v}{r^3}\frac{\omega}{a}\right)J_1\left(\frac{\omega r}{a}\right)-\left(\frac{v}{r}\frac{\omega^2}{a^2}+\frac{v}{r^2}\frac{\omega}{a}-\frac{\omega^2}{a^2 r}\right)J_0\left(\frac{\omega r}{a}\right)\right)}{\left(\frac{v}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)\left(I_1\left(\frac{\omega r}{a}\right)J_0\left(\frac{\omega r}{a}\right)-J_1\left(\frac{\omega r}{a}\right)I_0\left(\frac{\omega r}{a}\right)\right)}$$
$$+\left(\left(\frac{v}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)J_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2}J_0\left(\frac{\omega r}{a}\right)\right)$$
$$\times\frac{\left(\left(\frac{\omega^3}{a^3}-\frac{2}{r^2}\cdot\frac{\omega}{a}+\frac{v}{r^2}\frac{\omega}{a}\right)I_1\left(\frac{\omega r}{a}\right)-\left(\frac{v}{r}\frac{\omega^2}{a^2}+\frac{v}{r^2}\frac{\omega}{a}-\frac{\omega^2}{a^2 r}\right)I_0\left(\frac{\omega r}{a}\right)\right)}{\left(\frac{v}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)\left(I_1\left(\frac{\omega r}{a}\right)J_0\left(\frac{\omega r}{a}\right)-J_1\left(\frac{\omega r}{a}\right)I_0\left(\frac{\omega r}{a}\right)\right)}. \tag{3.6}$$

On Fig. 4. The function schedule is presented $\eta(\omega)$. On the schedule two areas of a spectrum are represented.

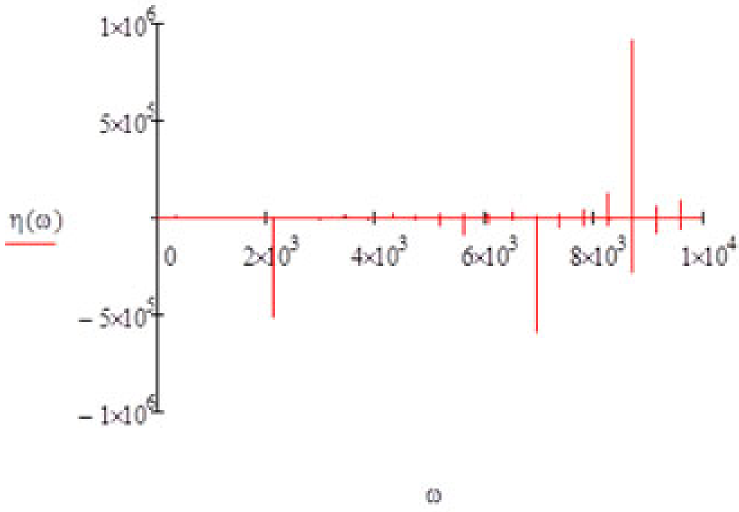

Fig.4.

4 The plate on a contour keeps horizontal position and is elastic leans.

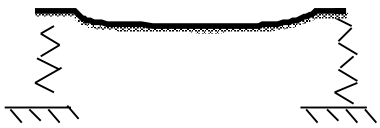

Fig.5.

The specified conditions look like:

$$\left.\frac{\partial W}{\partial r}\right|_{r=r_0}=0; \quad D\left(\frac{\partial^3 W}{\partial r^3}+\frac{1}{r}\cdot\frac{\partial^2 W}{\partial r^2}\right)=\ell W; \tag{4.1}$$

Substituting (2.8) in (4.1), we have

$$AJ_1\left(\frac{\omega r}{a}\right)+BI_1\left(\frac{\omega r}{a}\right)=0. \tag{4.2}$$

$$D\left(A\left[\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)J_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}J_0\left(\frac{\omega r}{a}\right)\right]\right.$$

$$\left.+B\left[\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)I_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}I_0\left(\frac{\omega r}{a}\right)\right]\right)=\ell W. \tag{4.3}$$

Here $W=AJ_0\left(\frac{\omega r_0}{a}\right)+BI_0\left(\frac{\omega r_0}{a}\right)$.
From (4.2)

$$A=-\frac{BI_1\left(\frac{\omega r}{a}\right)}{J_1\left(\frac{\omega r}{a}\right)}. \tag{4.4}$$

Substituting (4.4) in (4.3), we have

$$K(\omega)=\frac{\left[\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)J_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}J_0\left(\frac{\omega r}{a}\right)\right]\cdot I_i\left(\frac{\omega r}{a}\right)}{-I_1\left(\frac{\omega r}{a}\right)J_0\left(\frac{\omega r}{a}\right)+I_0\left(\frac{\omega r}{a}\right)J_1\left(\frac{\omega r}{a}\right)}$$

$$-\frac{\left[\frac{\omega}{a}\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)I_1\left(\frac{\omega r}{a}\right)+\frac{\omega^2}{a^2 r}I_0\left(\frac{\omega r}{a}\right)\right]J_1\left(\frac{\omega r}{a}\right)}{I_1\left(\frac{\omega r}{a}\right)J_0\left(\frac{\omega r}{a}\right)-I_0\left(\frac{\omega r}{a}\right)J_1\frac{\omega r}{a}}. \qquad (4.5)$$

The function schedule $K(\omega)-\omega$ i.e. a spectrum of frequencies is presented on Fig. 6. Here $K(\omega)=\frac{\ell(\omega)}{D}$

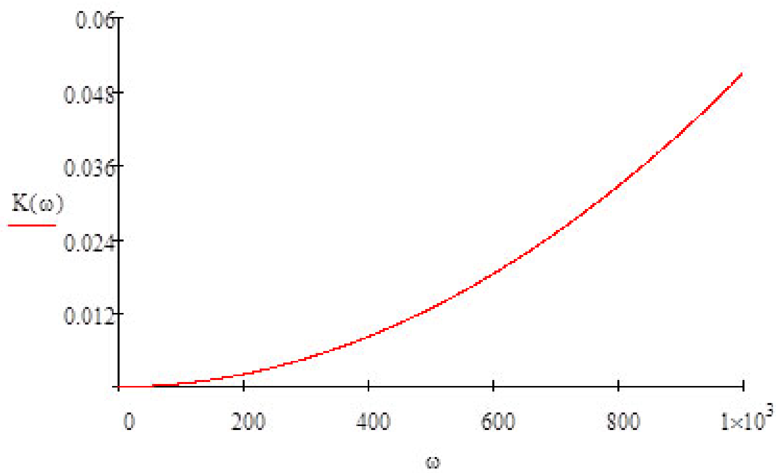

Fig.6.

5 On a plate contour there is a thickening, i.e. additional weight; it is necessary to consider it at drawing up of a boundary condition.

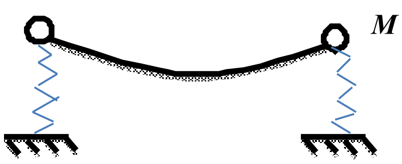

Fig.7.

Let's consider a case of hinge fastenings and weight of M,

$$\frac{\partial^2 W}{\partial r^2}+\frac{\nu}{r}\cdot\frac{\partial W}{\partial r}=0; \qquad (5.1)$$

Elastic supporting gives

$$M\omega^2 W = q\frac{\partial^3 W}{\partial r^3} + \frac{\nu}{r}\cdot\frac{\partial^2 W}{\partial r^2} - \frac{\nu}{r}\cdot\frac{\partial W}{\partial r}. \tag{5.2}$$

Substituting (2.8) in (5.1) it is had

$$A\frac{\omega^2}{a^2}\left[\frac{a}{\omega r}J_1\left(\frac{\omega r}{a}\right) - J_0\left(\frac{\omega r}{a}\right)\right] + B\frac{\omega^2}{a^2}\left[\frac{a}{\omega r}I_1\left(\frac{\omega r}{a}\right) - I_0\left(\frac{\omega r}{a}\right)\right]$$
$$-\frac{\nu}{r}\frac{\omega}{a}AJ_1\left(\frac{\omega r}{a}\right) - \frac{\nu}{r}\frac{\omega}{a}BI_1\left(\frac{\omega r}{a}\right) = 0. \tag{5.3}$$

$$q\left[\frac{\omega}{a}\left(\frac{\omega^2}{a^2} - \frac{2}{r^2}\right)\left(AJ_1\left(\frac{\omega r}{a}\right) + BI_1\left(\frac{\omega r}{a}\right)\right) + \frac{\omega^2}{a^2 r}\left(AJ_0\left(\frac{\omega r}{a}\right) + BI_0\left(\frac{\omega r}{a}\right)\right)\right.$$
$$+\frac{\nu}{r}\cdot\left(\frac{\omega}{ar}\left(AJ_1\left(\frac{\omega r}{a}\right) + BI_1\left(\frac{\omega r}{a}\right)\right) - \frac{\omega^2}{a^2}\left(AJ_1\left(\frac{\omega r}{a}\right) + BI_1\left(\frac{\omega r}{a}\right)\right)\right)$$
$$\left.+\frac{\nu}{r}\cdot\frac{\omega}{a}\left(AJ_1\left(\frac{\omega r}{a}\right) + BI_1\left(\frac{\omega r}{a}\right)\right)\right] = M\omega^2\left(AJ_0\left(\frac{\omega r}{a}\right) + BI_0\left(\frac{\omega r}{a}\right)\right). \tag{5.4}$$

From (5.3)

$$A = -\frac{B\cdot\left(\left(\frac{\nu}{r}\frac{\omega}{a} - \frac{\omega}{ar}\right)I_1\left(\frac{\omega r}{a}\right) + \frac{\omega^2}{a^2}I_0\left(\frac{\omega r}{a}\right)\right)}{\left(\frac{\nu}{r}\frac{\omega}{a} - \frac{\omega}{ar}\right)J_1\left(\frac{\omega r}{a}\right) + \frac{\omega^2}{a^2}J_0\left(\frac{\omega r}{a}\right)}. \tag{5.5}$$

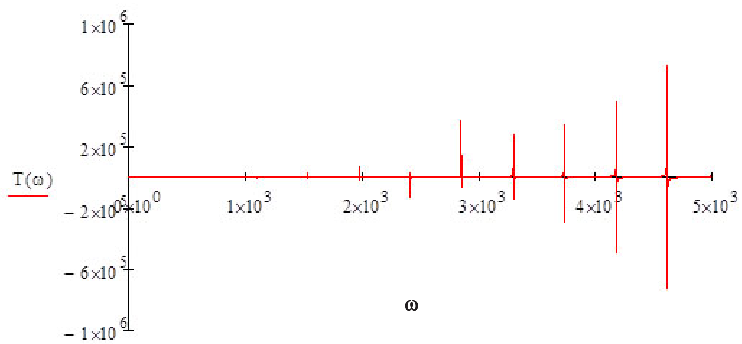

Fig.8.

Substituting (5.5) in (5.4), we have

$$T(\omega) = \left(\left(\frac{v}{r}\frac{\omega}{a} - \frac{\omega}{ar}\right)I_1\left(\frac{\omega r}{a}\right) + \frac{\omega^2}{a^2}I_0\left(\frac{\omega r}{a}\right)\right)$$
$$\times\frac{\left(\left(\frac{\omega}{a}\cdot\left(\frac{\omega^2}{a^2} - \frac{2}{r^2}\right) + \frac{v}{r}\frac{\omega}{ar} + \frac{v}{r}\frac{\omega}{a}\right)J_1\left(\frac{\omega r}{a}\right) + \left(\frac{\omega^2}{a^2 r} - \frac{v}{r}\frac{\omega^2}{a^2}\right)J_0\left(\frac{\omega r}{a}\right)\right)}{\left(\frac{v}{r}\frac{\omega}{a} - \frac{\omega}{ar}\right)\left(I_1\left(\frac{\omega r}{a}\right)J_0\left(\frac{\omega r}{a}\right) - J_1\left(\frac{\omega r}{a}\right)I_0\left(\frac{\omega r}{a}\right)\right)}$$
$$-\left(\frac{v}{r}\frac{\omega}{a} - \frac{\omega}{ar}\right)J_1\left(\frac{\omega r}{a}\right) + \frac{\omega^2}{a^2}J_0\left(\frac{\omega r}{a}\right)$$

$$\times \frac{\left(\left(\frac{\omega}{a}\cdot\left(\frac{\omega^2}{a^2}-\frac{2}{r^2}\right)+\frac{v}{r}\frac{\omega}{ar}+\frac{v}{r}\frac{\omega}{a}\right)I_1\left(\frac{\omega r}{a}\right)+\left(\frac{\omega^2}{a^2 r}-\frac{v}{r}\frac{\omega^2}{a^2}\right)I_0\left(\frac{\omega r}{a}\right)\right)}{\left(\frac{v}{r}\frac{\omega}{a}-\frac{\omega}{ar}\right)\left(I_1\left(\frac{\omega r}{a}\right)J_0\left(\frac{\omega r}{a}\right)-J_1\left(\frac{\omega r}{a}\right)I_0\left(\frac{\omega r}{a}\right)\right)}. \tag{5.6}$$

The spectrum of frequencies of free fluctuations is presented on Fig. 8.

Here $T(\omega) = \frac{M(\omega)}{q}$

Spectra of free fluctuations for various chances of fastening of round plates are presented.

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