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Study of vibrations of stiffened skin orthotropic cylindrical shell filled with viscous fluid

Fuad S. Latifov · Konul A. Novruzova

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Abstract. *The paper is devoted to the solution of a problem of free oscillations of laterally strengthened orthotropic cylindrical shell filled with viscous fluid, by variational principle. The frequency equation of oscillations of a laterally strengthened orthotropic cylindrical shell filled with viscous fluid is constructed on the basis of Ostrogradsky-Hamilton's variational principle and is realized numerically. Surface loads acting on a laterally strengthened cylindrical shell as viewed from fluid are determined from the solutions of Navier-Stock's linearized equation.*

Keywords. oscillations, shell, ideal liquid, stress, viscous fluid, strengthening, variation principle.

Mathematics Subject Classification (2010): 74J05

1 Introduction

Structural materials are widely used in different fields of machine building, aircraft industry, shipbuilding, etc. This reduces to necessity of total account of properties of materials and constructions for more rational designing and reliable strength analysis. For more complete description of load-bearing capacity of a construction, it is expedient to take into account the external action forces as viewed from medium. One of such actions is its contact with viscous fluid. External action forces as viewed from viscous fluid, in fact are surface forces and are stipulated by the contact between the shell and viscous fluid. The solution of such type problems represents mathematical difficulty that deepens with account of dynamical effects that is necessary in problems of seismic stability, vibration that are often encountered in engineering. Therefore, development of an approximate method is required. One of the approximate methods is variational. This is explained by the fact that it allows to get consistent approximate theories of thin walled medium-contacting constructions of shells and bars type.

Note that the solutions described in references belong chiefly to a strengthened mediumless cylindrical shell [1]. Oscillations of smooth isotropic cylindrical shells with medium were completely studied in the papers [2,3]. Behavior of deformable smooth shells with flowing fluid was considered in the monograph

F.S. Latifov, K.A. Novruzova
Institute of Mathematics and Mechanics of NAS of Azerbaijan
9, B.Vahabzade str., AZ 1141, Baku, Azerbaijan
E-mail: flatifov@mail.ru

[4]. In the paper [5], the oscillations of laterally strengthened orthotropic Shells with flowing ideal fluid in medium are studied. Eigen oscillations of a flowing fluid-filled isotropic cylindrical shell strengthened with crossed system of ribs in infinite elastic medium was considered in the paper [6]. Free oscillations of ideal fluid-dilled ribbed isotropic cylindrical shells at axial compression were studied in [7]. As it follows from the cited review, there are the papers devoted to free oscillations of compressible viscous fluid-filled anisotropic cylindrical shells. Therefore study of one of main dynamical characteristics of the elastic system the frequency of eigen oscillations of viscous fluid-filled cylindrical shells of great practical value.

A ribbed shell is considered as a system consisting of an eigen anisotropic shell and lateral ribs rigidly connected with it along the contact lines. It is accepted that the stress-strain state of a shell may be completely determined within the linear theory of thin shells based on Kirchoff-Liav hypothesis, while for calculation of ribs, the theory of Kirchhoff-Klebsh curvilinear bars is applircable. The system of coordinates is chosen so that coordinate lines coincide with the lines of the main curvature of the median surface of the shell. Therewith it is supposed that the ribs are located along the coordinate lines, and their edges as the edge of the casing lie on the same coordinate plane. The strain state of the casing may be determined by three components of displacements of its median surface u, ϑ and w. Therewith the turning angle of normal elements φ_1, φ_2 with respect to coordinate lines y and x are expressed by w and ϑ by means of dependences $\varphi_1 = -\frac{\partial w}{\partial x}$, $\varphi_2 = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R}\right)$, where R is the radius of the shell's median surface.

For describing strain state of ribs, in addition to three components of displacements of gravity centers of their cross sections (u_j, ϑ_j, w_j of the j-th cross section), it is necessary to define also the twisting angles φ_{kpi} and φ_{kpj}.

Taking account that according to accepted hypotheses it holds the constancy of radial deflections along the height of sections, and also equality of appropriate twisting angles following from the condition of rigid connection of ribs and a shell, we write the following relations:

$$u_j\left(y\right) = u\left(x_j, y\right) + h_j\varphi_1\left(x_j, y\right); \vartheta_j\left(x\right) = \vartheta\left(x_j, y\right) + h_j\varphi_2\left(x_j, y\right);$$

$$w_j\left(x\right) = w\left(x_j, y\right); \varphi_1 = \varphi_2\left(x_j, y\right); \varphi_{kpj}\left(x\right) = \varphi_1\left(x_j, y\right); \tag{1}$$

Here $h_i = 0,5h + H_i^1$, $h_j = 0,5h + H_j^1$, h is the shell's thickness, H_i^1 and H_j^1 are distances from the axes of the i-th longitudinal and j-th lateral bars to the shell's surface, x_i and y_i are the coordinates of junction lines of ribs with a shell, φ_i, φ_{kpi} and φ_j, φ_{kpj} are turning and twisting angles of lateral sections of longitudinal and lateral bars, respectively.

For external actions it is supposed that surface loads acting on a ribbed shell as viewed from viscous fluid may be reduced to the components q_x, q_y and q_z applied to the median surface of the shell.

We get differential equations of motion and natural boundary conditions for a laterally strengthened viscous fluid-filled orthotropic cylindrical shell based on Ostrogradsky-Hamilton's variational principle. For that we preliminarily write potential and kinetic energy of the system.

The potential energy of elastic deformation of an orthotropic cylindrical shell is of the form:

$$\Pi_0 = \frac{hR}{2}\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}\left\{b_{11}\left(\frac{\partial u}{\partial x}\right)^2 - 2\left(b_{11}+b_{12}\right)\frac{w}{R}\frac{\partial u}{\partial x}\right.$$

$$+\frac{w}{R^2}\left(b_{11}+2b_{12}+b_{22}\right) + b_{22}\left(\frac{\partial \vartheta}{\partial y}\right)^2 - 2\left(b_{12}+b_{22}\right)\frac{w}{R}\frac{\partial \vartheta}{\partial y} + 2b_{12}\frac{1}{R}\frac{\partial u}{\partial x}\frac{\partial \vartheta}{\partial y}$$

$$\left.+b_{66}\left(\frac{\partial u}{\partial y}\right)^2 + b_{66}\left(\frac{\partial \vartheta}{\partial x}\right)^2 + 2b_{66}\frac{\partial u}{\partial y}\frac{\partial \vartheta}{\partial x}\right\}dxdy, \tag{2}$$

where $b_{11} = \frac{E_1}{1-\nu_1\nu_2}$; $b_{22} = \frac{E_2}{1-\nu_1\nu_2}$; $b_{12} = \frac{\nu_2 E_1}{1-\nu_1\nu_2} = \frac{\nu_1 E_2}{1-\nu_1\nu_2}$; $b_{66} = G_{12} = G$, R is he radius of shell's median surface, h is the shell's thickness u,ϑ, w are the components of displacements of the points of the shell's median surface, E_1, E_2 are module of elasticity of the shell's material in coordinate directions, G is the modulus of elasticity at shear.

The expressions for potential energy of elastic deformation of the j-th lateral rib are as follows [1]:

$$\Pi_j = \frac{1}{2}\int\limits_{y_1}^{y_2}\left[\widetilde{E}_j F_j\left(\frac{\partial\vartheta_j}{\partial y} - \frac{w_j}{R}\right)^2 + \widetilde{E}_j J_{xj}\left(\frac{\partial^2 w_j}{\partial x^2} - \frac{w_j}{R^2}\right)^2\right.$$

$$\left.+\widetilde{E}_j J_{zj}\left(\frac{\partial^2 u_i}{\partial y^2} - \frac{\varphi_{kpj}}{R}\right)^2 + \widetilde{G}_j J_{kpj}\left(\frac{\partial\varphi_{kpi}}{\partial y} + \frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right] dy. \tag{3}$$

Here F_j, J_{zj}, J_{yj}, J_{kpj} are the area and inertia moments of the cross section of the j-th lateral bar with respect to the axis Oz and the axis parallel to the axis Oy and passing through the gravity center of the section and also its inertia moment at torsion; $\widetilde{E}_j$, $\widetilde{G}_j$ are elasticity and shear module of the material of the j-th lateral bar.

Potential energy of external surface loads acting is viewed from viscous fluid applied to the shell, is defined as a work done by these loads when taking the system from the deformed state to the initial undeformated one and is represented in the form:

$$A_0 = -\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}(q_x u + q_y\vartheta + q_z w)\,dxdy. \tag{4}$$

Total potential energy of the system equals the sum of potential energies of elastic deformations of the shell and lateral ribs, and also potential energies of all external loads acting as viewed from viscous fluid

$$\Pi = \Pi_0 + \sum_{j=1}^{k_2}\Pi_j + A_0. \tag{5}$$

Kinetic energies of the shell and lateral ribs are written in the form [1]:

$$K_0 = \rho h\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial\vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] dxdy$$

$$K_j = \rho_j F_j\int\limits_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2 + \left(\frac{\partial\vartheta_j}{\partial t}\right)^2 + \left(\frac{\partial w_j}{\partial t}\right)^2 + \frac{J_{kpj}}{F_j}\left(\frac{\partial\varphi_{kpj}}{\partial t}\right)^2\right] dy. \tag{6}$$

Here t is a time coordinate, ρ, ρ_j are densities of materials from which the shell was made, j is the lateral bar. Kinetic energy of the laterally strengthened shell

$$K = K_0 + \sum_{j=1}^{k_2} K_j. \tag{7}$$

The equations of motion of a ribbed shell are obtained based on Ostrogradsky-Hamilton's action stationarity principle:

$$\delta W = 0, \tag{8}$$

where $W = \int\limits_{t'}^{t''}\widetilde{L}dt$ is Hamilton's action, $\widetilde{L} = K - \Pi$ is Langrange's function, t'' and t' are the given arbitrary times.

Supposing that the shell is strengthened with infinitely many number of ribs, by limit passage $k_2 \to \infty$ allowing for (1) and that the variation and differentiation operations are permutational, we can reduce equation (8) to the form

$$\left[b_{11}\frac{\partial^2}{\partial\xi^2} + b_{66}\frac{\partial^2}{\partial\theta^2}\right]u + (b_{12} + b_{66})\frac{\partial^2\vartheta}{\partial\xi\partial\theta} - (b_{11} + b_{12})\frac{\partial w}{\partial\xi} = \frac{R^2 q_x}{2h}$$

$$(b_{12}+b_{66})\frac{\partial^2 u}{\partial\xi\partial\theta}+\left\{b_{66}\frac{\partial^2}{\partial\xi^2}+\left[G_{12}+\left(1-\frac{h_j}{R}\right)^2 G_{12}\gamma_j^{(2)}+b_{22}\right]\frac{\partial^2}{\partial\theta^2}\right\}\vartheta$$

$$-\left[b_{12}+b_{22}+\left(1-\frac{h_j}{R}\right)^2 G_{12}\gamma_j^{(2)}\right]\frac{\partial w}{\partial\theta}=\frac{R^2 q_y}{2h} \tag{9}$$

$$(b_{11}+b_{12})\frac{\partial}{\partial\xi}u+\left\{\left[b_{12}+b_{22}+\left(1-\frac{h_j}{R}\right)G_{12}\gamma_j^{(2)}\right]\frac{\partial}{\partial\theta}\right.$$

$$\left.-\left(1-\frac{h_j}{R}\right)\delta_j^{(2)}G_{12}\frac{\partial^3}{\partial\theta^3}\right\}\vartheta+$$

$$+\left\{b_{11}+2b_{12}+b_{22}+\left(\gamma_j^{(2)}+\eta_{j1}^{(2)}\right)G_{12}+2\left(\delta_j^{(2)}+\eta_{j1}^{(2)}\right)G_{12}\frac{\partial^2}{\partial\theta^2}\right.$$

$$+a^2\left[b_{11}\frac{\partial^4}{\partial\xi^4}+2\left(b_{11}+b_{12}\right)\frac{\partial^4}{\partial\xi^2\partial\theta^2}\right.$$

$$\left.+\left(b_{12}+\eta_{j1}^{(2)}G_{12}+\eta_{j2}^{(2)}G_{12}\right)\frac{\partial^4}{\partial\theta^4}\right\}w=\frac{R^2}{2h}q_z$$

$$a^2=\frac{h^2}{12R^2},\xi=\frac{x}{R},\gamma_j^{(2)}=\frac{E_j}{G_l}\overline{\gamma}_j^{(2)},\overline{\gamma}_j^{(2)}=\frac{F_j}{Lh},\overline{\delta}_j^{(2)}=\frac{h_j}{R}\overline{\gamma}_j^{(2)},\overline{\eta}_j^{(2)}=\left(\frac{h_j}{R}\right)^2\overline{\gamma}_j^{(2)},$$

$$\delta_j^{(2)}=\frac{h_s}{R}\overline{\gamma}_j^{(2)},\eta_{j2}^{(2)}=\frac{\widetilde{E}_j}{\widetilde{G}_j}\overline{\eta}_j^{(2)},\overline{\eta}_{j1}^{(2)}=\frac{\widetilde{E}_j J_{xj}}{\widetilde{G}_j LhR^2},\theta=\frac{y}{R},\overline{\mu}_j^{(2)}=\frac{J_{kpj}}{LhR^2}.$$

Surface loads q_x, q_y and q_z acting on longitudinally strengthened shell as viewed from viscous fluid, are determined from the solutions of Navier-Stock's linearized equation:

$$\rho_0\frac{\partial\overrightarrow{\vartheta}}{\partial t}=-gradp-\frac{\overline{\mu}}{3\rho_0 a_*^2}qrad\left(\frac{\partial p}{\partial t}\right)+\overline{\mu}\nabla^2\overrightarrow{\vartheta}, \tag{10}$$

where $\overline{\mu}$ is dynamical coefficient of viscosity, p is pressure at some point of fluid, ρ_0 is fluid's density, a_* is sound velocity in fluid, ∇^2 is the Laplace operator, $\overrightarrow{\vartheta}\ (\vartheta_x,\vartheta_y,\vartheta_z)$ is the velocity vector of arbitrary point of fluid.

On the contact surface a shell-viscous fluid the following one is fulfilled $(r=R)$:

$$\vartheta_x=\frac{\partial u}{\partial t},\vartheta_y=\frac{\partial\vartheta}{\partial t},\vartheta_r=\frac{\partial w}{\partial t}. \tag{11}$$

$$q_x=-\sigma_{rx},q_\theta=-\sigma_{r\theta},q_z=-p \tag{12}$$

where the viscosity forces are determined by the equalities

$$\sigma_{rx}=\overline{\mu}\left(\frac{\partial\vartheta_z}{\partial x}+\frac{\partial\vartheta_x}{\partial z}\right),\sigma_{r\theta}=\overline{\mu}\left(\frac{\partial\vartheta_z}{\partial y}+\frac{\partial\vartheta_y}{\partial z}\right). \tag{13}$$

By means of discontinuity equation and the equation of state, equation (10) is reduced to the equation with respect to p:

$$\frac{1}{a^2}\frac{\partial^2 p}{\partial t^2}=\nabla^2 p+\frac{4}{3}\frac{\overline{\mu}}{\rho_0 a^2}\frac{\partial p}{\partial t}. \tag{14}$$

Then we consider the hingely supported shells, i.e. for $\xi=0$ and $\xi=\xi_1$ $(\xi_1=L/R)$ the following boundary conditions are fulfilled:

$$\vartheta=w=0,\ \ T_1=M_1=0.$$

We look for the components of the displacement vector of the point of the shell's median surface in the form

$$u=u_0\cos n\theta\cos\frac{m\pi}{\xi_1}\xi\sin\omega_1 t_1;\ \ \vartheta=\vartheta_0\sin n\theta\sin\frac{m\pi}{\xi_1}\xi\sin\omega_1 t_1;$$

$$w = w_0 \cos n\theta \sin \frac{m\pi}{\xi_1} \xi \sin \omega_1 t_1, \tag{15}$$

where u_0, ϑ_0, w_0 are unknown constants, $t_1 = \omega_0 t$, $\omega_0 = \sqrt{\frac{G_{12}}{(1-\nu^2)\rho R^2}}$, $\omega_1 = \omega/\omega_0$, ω is a sought-for frequency.

After separation of variables, the solution of equation (14) has the form

$$p = p_0 J(\lambda r) \cos n\theta \sin \frac{m\pi}{\xi_1} \xi \sin \omega t. \tag{16}$$

Using (16) and (10) we can define the velocity components in fluid and by formula (13) the viscosity forces.

Complementing by contact conditions (11), (12) the system of equations of motion of the shell (9), fluid (10) we arrive at the contact problem on oscillations of viscous fluid-filled orthotropic shell strengthened with lateral ribs. In other words, the problem of oscillations of viscous fluid-filled orthotropic shell is reduced to joint integration of equations of theory of shells, fluid, subject to the indicated conditions on their contact surface.

Using and (11)-(13), (15) and (16) and (9) the problem is reduced to the homogeneous system of linear algebraic equations of third order

$$a_{i1} u_0 + a_{i2} \vartheta_0 + a_{i3} w_0 = 0 \quad (i = 1, 2, 3). \tag{17}$$

The elements a_{i1}, a_{i2}, a_{i3} $(i = 1, 2, 3)$ have a bulky form and we don't cite them here. The nontrivial solution of the system of linear algebraic equations of third order is possible only in the case when (17) is the root of its determinant. The definition of ω_1 is reduced to a transcendental equation as ω_1 is contained in the arguments of the Bessel function J_n:

$$\det \left\| a_{ij} \right\| = 0. \tag{18}$$

Note that for $\bar{\mu} = 0$ equation (18) goes into frequency equation of free oscillations of a laterally strengthened ideal fluid-filled orthotropic cylindrical shell.

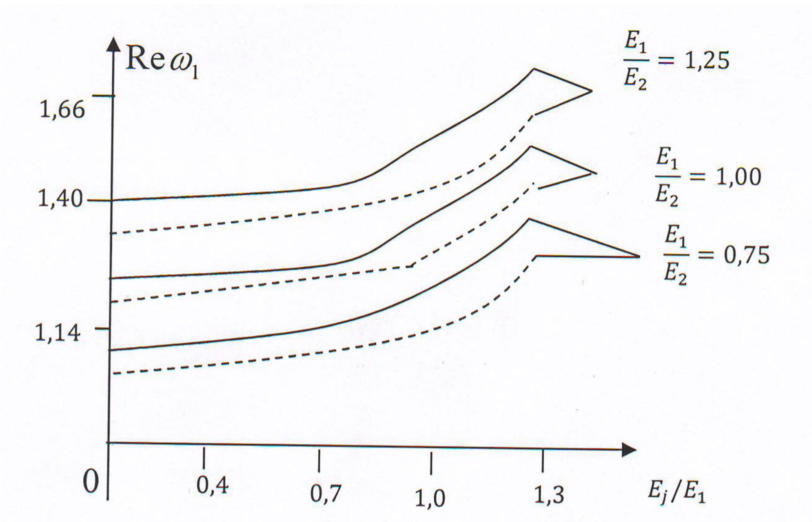

Fig.1. Dependence of the oscillations frequency parameter on elasticity modulus of lateral bar. The dotted line is viscous fluid: the solid line-no viscosity.

Let us consider some results of calculations performed proceeding from the above dependences in ICM.

For geometrical and physical parameters characterizing the material of the shell, fluid and longitudinal bar, we accepted:

$$h^* = \frac{h}{R} = 0,25 \cdot 10^{-2}; \xi_1 = 1; E_j = 6,67 \cdot 10^9 n/m^2; \nu = 0,3; h_j = 1,39mm;$$

$$R = 160mm; L = 800mm; F_j = 5,75mm^2; J_{xj} = 19,9mm^4; J_{kp,j} = 0,48mm^4;$$

$$\rho_0/\rho = 0,105; \nu_2 = 0,19; \nu_1 = 0,11; a_* = 1350\frac{m}{\sec}; \overline{\mu} = 10,02\frac{kg}{\sec m}.$$

Dependence of the frequency parameter on elasticity modulus of the lateral bar for different ratios of elasticity modulus of the shell material are depicted in Fig. 1. Calculation shows that account of fluid's material viscosity reduces to decrease of frequency of eigen oscillations of the system in comparison when the fluid is ideal. Furthermore, with increasing the ratios E_j/E_1 the frequency of eigen oscillations of the system at first smoothly and then sharphy increase. Furthermore, with increasing the ratios, the frequencies of eigen oscillations of the system increase. The dependence of frequency parameter on density of lateral bar for different ratios of elasticity module of the shell material are given in Fig. 2. It is seen from the figure that with increasing the ratio of frequencies ρ_i/ρ eigen oscillations of the system decreases. In the same place, with increasing the ratios $\frac{E_1}{E_2}$ the frequencies of eigen oscillations of the system increase.

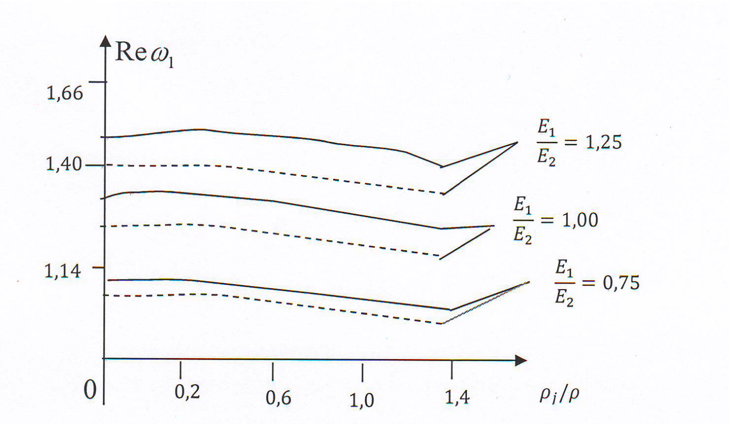

Fig. 2. Dependence of frequency parameter of oscillations on density of lateral bar. The dotted line is fluid. The solid line is no viscosity.

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