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Behavior of rotating cylinder under thermomechanical loadings

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Abstract. *We consider the solution of a problem on behavior of a hollow annular section viscoelastic cylinder rotating around its symmetry axis, connected with an elastic shell with uniform pressure and high temperature (i.e. combustion temperature) acting on its internal surface. Mechanical properties of the material are temperature dependent and this transforms the problem into nonlinear one. A moving nonlinear boundary value problem with regard to ablation on the internal surface is solved. A new method for solving a nonlinear problem of linear thermoviscoelasticity with arbitrary rheology is suggested and this reduces to solving Volterra-type integral equation of second kind.*

Keywords. viscoelastic cylinder · ablation · mechanical properties · dependence on temperature · nonlinear equation · Volterra type integral equation.

Mathematics Subject Classification (2010): 74B20

1 Introduction

Determination of thermomechanical stresses in viscoelastic cylinders is of great technical interest in conformity to the problem of determination of stresses arising in grains of solid fuels. At present, this topic is one of the intensely studied ones in theory of viscoelasticity. The brief review of preceding papers is in [6-9].

We consider an annular section cylinder rotating around its symmetry axis with angular velocity $\omega(t)$. Uniformly distributed loads act on the internal and external surfaces of the cylinder. Temperature T_1, that in the sequel is maintained constant, instantly influences on the internal surface.

Under ablation of the internal surface, the radius r_1 is a monotonically increasing time function $(r_1 = r_1(t))$, i.e. a moving boundary problem is studied. The external surface of the cylinder is rigidly connected with an elastic shell of thickness h, as it was shown in [6] by using momentless theory one can find:

$$\sigma_r(b,t) = -Be_\theta(b,t) \tag{1.1}$$

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$$B = \frac{1 - v_R}{1 + v_R} \cdot \frac{E_R h}{h + b},$$

where E_R, v_R are elastic characteristics of the shell; b is an inner radius, h is the shell's thickness.

In the case under consideration, only one of the vector displacements components is nonzero. For deformations tensor vector components

$$u = u\left(r, t\right) \neq 0 e_r = \frac{\partial u}{\partial r}; \quad e_\theta = \frac{u}{r}.$$

Volume deformation is determined by the formula

$$\frac{\partial u}{\partial r} + \frac{u}{r} = l\left(r, t\right) \tag{1.2}$$

or

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\right)\left(ur\right) = l,$$

hence

$$u\left(r, t\right) = A\left(t\right)/2r + \left({}^1\!/_r\right) \int rl\left(r, t\right) dr$$

$$e_r = -A\left(t\right)/2r^2 + l\left(r, t\right) - \frac{1}{r^2} \int rl\left(r, t\right) dr \tag{1.3}$$

$$e_\theta = A\left(t\right)/2r^2 + \frac{1}{r^2} \int rl\left(r, t\right) dr.$$

In isotropic incompressible linear-viscoelastic materials, the stresses and deformations, allowing for temperature dependence of the material properties are connected as follows:

$$\sigma_{r,\theta} = \int_0^t J\left(t' - \tau'\right) \frac{\partial}{\partial \tau} l\left(r, t\right) d\tau + \int_0^t K\left(t' - \tau'\right) \frac{\partial}{\partial \tau} e_{r,\theta}\left(\tau\right) d\tau$$

$$\sigma_z = \int_0^t J\left(t' - \tau'\right) \frac{\partial}{\partial \tau} l\left(r, \tau\right) d\tau, \tag{1.4}$$

where J, K are the functions of volume and shear relaxation, t' is the reduced time.

Using temperature time analogs [1] for describing the reduced time, and using the technique for solving such problems, we can represent:

$$\sigma_{r,\theta} = \sum_{n=0}^{\infty} \lambda^n \sigma_{r,\theta}^{(n)}; \qquad e_{r,\theta} = \sum_{n=0}^{\infty} \lambda^n e_{r,\theta}^{(n)},$$

$$J\left(t' - \tau'\right) = \sum_{n=1}^{\infty} \lambda^n J_n\left(r, t - \tau\right) + J_0\left(t - \tau\right) \tag{1.5}$$

$$K\left(t' - \tau'\right) = \sum_{n=1}^{\infty} \lambda^n K_n\left(r, t - \tau\right) + K_0\left(t - \tau\right), u_n\left(r, t\right) = \frac{1}{r} \int rl_n\left(r, t\right) dr,$$

$$l\left(r, t\right) = \sum_{n=0}^{\infty} \lambda^n l_n\left(r, t\right), u_0\left(r, t\right) = A\left(t\right)/2r + \left({}^1\!/_r\right) \int rl_0\left(r, t\right) dr,$$

where λ is a small parameter, the expressions for $J_n,\ K_n$ are reduced in [9]. Taking the last expressions into account and equating the same powers of λ for $\sigma_{r,\theta}^{(0)}$, we have:

$$\sigma_{r,\theta}^{(0)} = \int_0^t J_0\left(t - \tau\right) \frac{\partial}{\partial \tau} l_0\left(r, t\right) d\tau + \int_0^t K_0\left(t - \tau\right) \frac{\partial}{\partial \tau} e_{r,\theta}^{(0)}\left(\tau\right) d\tau$$

$$\sigma_z = \int_0^t J_0\left(t - \tau\right) \frac{\partial}{\partial \tau} l_0\left(r, \tau\right) d\tau. \tag{1.6}$$

The stress tensor components satisfy the equation:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \rho\omega^2(t) \cdot r = 0$$

$$\int_0^t [J_0(t-\tau) + K_0(t-\tau)] \frac{\partial^2}{\partial r \partial \tau} l(r,\tau)\, d\tau + \rho\omega^2(t) \cdot r = 0. \tag{1.7}$$

For convenience we introduce the denotation:

$$\Omega_0(t) = \int_0^t H_0(t-\tau) \frac{\partial}{\partial \tau} \omega^2(\tau)\, d\tau, \tag{1.8}$$

where the auxiliary function $H_0(t)$ is the solution of the equation:

$$\int_0^t [J_0(t-\tau) + K_0(t-\tau)] \frac{\partial}{\partial \tau} H_0(\tau)\, d\tau = 1. \tag{1.9}$$

Transforming equation (1.7) we get:

$$l_0(r,t) = l_0(t) - \frac{1}{r}\rho\Omega_0(t) r^2. \tag{1.10}$$

Then:

$$e_r^{(0)} = l_0(t)/2 + A(t)/2r^2 - \frac{3}{8}\rho r^2 \Omega_0(t) \tag{1.11}$$

$$e_\theta^{(0)} = l_0(t)/2 + A(t)/2r^2 - \frac{\rho r^2}{8}\Omega_0(t)\, \sigma_r^{(0)}$$

$$= \int_0^t [J_0(t-\tau) + K_0(t-\tau)] \frac{\partial}{\partial \tau} l_0(\tau)\, d\tau - \frac{1}{2r^2}\int_0^t K_0(t-\tau)\frac{\partial}{\partial r} A(\tau)\, d\tau$$

$$-\frac{1}{r}\rho r^2 \left\{\omega^2(t) - \frac{1}{4}\int_0^t K_0(t-\tau)\frac{\partial \Omega_0(\tau)}{\partial \tau}\partial\tau\right\} \tag{1.12}$$

$$\sigma_\theta^{(0)} = \int_0^t \left[J_0(t-\tau) + \frac{1}{r}K_0(t-\tau)\right] \frac{\partial}{\partial \tau} v_0(\tau)\, d\tau + \frac{1}{2r^2}\int_0^t K_0(t-\tau)\frac{\partial}{\partial r} A(\tau)\, d\tau$$

$$-\frac{1}{r}\rho r^2 \left\{\omega^2(t) - \frac{1}{4}\int_0^t K_0(t-\tau)\frac{\partial}{\partial \tau}\Omega_0(\tau)\,\partial\tau\right\}$$

$$\sigma_z^{(0)} = \int_0^t J_0(t-\tau)\frac{\partial}{\partial \tau} l_0(\tau)\, d\tau - \frac{1}{r}\rho r^2 \int_0^t J_0(t-\tau)\frac{\partial}{\partial \tau}\Omega_0(\tau)\, d\tau.$$

Here we have two unknown functions $l_0(t)$ and $A(t)$, that should be determined from the boundary conditions.

$$\sigma_r^{(0)}[a(t), t] = -\pi(t), \tag{1.13}$$

where $r_1 = a(t)$ is change of the radius of the internal surface in the availability of ablation that is a monotonically increasing time function.

$\pi(t)$ is the given pressure on the internal surface. The external surface of the viscoelastic cylinder is rigidly connected with an elastic shell of thickness h, and the boundary condition for $r = r_2 = b$ will be in the form (1.1). Satisfying boundary conditions (1.1), (1.12), (1.13) after some transformations we get:

$$\int_0^t [J_0\ (t-\tau) - K_0\ (t-\tau)] \frac{\partial}{\partial \tau} l_0(r_2,\tau)\, d\tau$$

$$\int_0^t [K(t-\tau) - B] \cdot \frac{\partial}{\partial \tau}\left[\frac{l_0(r_2,\tau)}{2} + \frac{A(\tau)}{2b^2}\right] d\tau$$

$$-\pi(t) = \int_0^t [J_0\ (t-\tau) + \frac{1}{r}K_0\ (t-\tau)] \frac{\partial}{\partial \tau} l_0(r_2,\tau)\, d\tau$$

$$-\frac{1}{2a^2(t)}\int_0^t K_0(t-\tau)\frac{\partial}{\partial\tau}A(\tau)\,d\tau.$$

Excepting the terms containing the integrals with $l_0(r_2,t)$, we get

$$\int_0^t K_0(t-\tau)\frac{\partial}{\partial\tau}A(\tau)\,d\tau=\frac{Ba^2(t)\,\xi_0(t)}{a^2(t)-b^2}-\frac{2a^2(t)\,b^2P_0(t)}{a^2-b^2},$$

where

$$P(t)=\pi(t)+\frac{B\rho b^2\Omega_0(t)}{8}-N_2\cdot\left(a^2(t)-b^2\right)$$

$$N_2(t)=\frac{1}{2}\rho\left(\omega^2(t)-\frac{1}{4}\int_0^t K_0(t-\tau)\frac{\partial}{\partial\tau}\Omega_0(\tau)\,d\tau\right)$$

$P(t),\ N_2(t)$ are the known functions.

From these equations excluding the integrals containing the terms with $A(t)$, we get:

$$\int_0^t\left[J_0\ (t-\tau)+\frac{1}{r}K_0\ (t-\tau)\right]\frac{\partial}{\partial\tau}l_0(\tau)\,d\tau=\frac{B\xi_0(t)}{2\,(a^2(t)-b^2)}-\frac{a^2P_1(t)}{a^2(t)-b^2},$$

where

$$P_1(t)=\pi(t)+\frac{B\rho b^4\Omega_0}{8a^2}-2\left(1-\frac{b^2}{a^2}\right)N_2(t),$$

$$\xi_0(t)=b^2l_0(t)+A(t).$$

Allowing for them, we rewrite (1.12)

$$\sigma_r^{(0)}=\frac{B\xi_0(t)}{2\,(a^2(t)-b^2)}\left(1-\frac{a^2(t)}{r^2}\right)-\frac{a^2(t)}{a^2(t)-b^2}\left(P_1(t)-\frac{b^2}{r^2}P_0(t)\right)-N_2(t)$$

$$\sigma_\theta^{(0)}=\frac{B\xi_0(t)}{2\,(a^2(t)-b^2)}\left(1+\frac{a^2(t)}{r^2}\right)-\frac{a^2(t)}{a^2(t)-b^2}\left(P_1+\frac{b^2}{r^2}P_0\right)-N_2(t).\qquad(1.14)$$

Introduce the auxiliary function $\Phi_0(t)$ satisfying the equation

$$\int_0^t\left[J_0\ (t-\tau)+\frac{1}{r}K_0\ (t-\tau)\right]\frac{\partial\Phi_0(\tau)}{\partial\tau}d\tau=\int_0^t(K_0(t-\tau)-B)\frac{\partial}{\partial\tau}\left(j_0(\tau)+\frac{1}{r}K(\tau)\right)d\tau.$$

From the last expressions we get

$$B\xi_0(t)-\left(\frac{a^2}{b^2}-1\right)\int_0^t\Phi_0(t-\tau)\frac{\partial}{\partial\tau}\xi_0(\tau)\,d\tau=2a^2(t)\,P_1(t)$$

$\xi_0(t)$ is determined from (1.14). Removing the discontinuity for $t=0$ and integrating by parts, we get

$$\xi_0(t)=\int_0^t\Phi^*(t,\tau)\,\xi_0(\tau)\,d\tau=\pi^*(t),\qquad(1.15)$$

where

$$\Phi^*=(t,\tau)=[1/\Phi_0(t)]\left[1-a^2/b^2\right]\frac{\partial}{\partial\tau}\Phi_0(t-\tau)$$

$$\pi^*(t)=[1/\Phi_0(t)]\left[2a^2(t)\cdot P_1(t)\right]$$

$$\Phi_0(t)=B=\left[1-a^2(t)/b^2\right]\Phi(0).$$

Equation (1.15), is Volterra's integral equation of second kind, whose general solution is of the form:

$$\xi_0(t)=\pi^*(t)+\int_0^t Q^*(t,\tau)\,\pi^*(\tau)\,d\tau,\qquad(1.16)$$

where Q^* is a resolvent of the equation.

By means of the iterations method we can find Q^*in the form

$$Q^*(t,\tau)=\sum_{n=1}^{\infty}q^{(n)}(t,\tau),$$

$$q^{(1)}(t,\tau)=\Phi^*(t,\tau)\,q^{(n)}(t,\tau)=\int_{\tau}^{t}q^{(1)}(t,s)\,q^{(n-1)}(t,s)\,ds,$$

$$\xi_0^{(1)}(t)=\pi^*(t)+\int_0^t\Phi^*(t,\tau)\,\pi^*(\tau)\,d\tau.$$

Introduce the function $G_0(t)$ satisfying the relation:

$$\int_0^t K_0(t-\tau)\frac{\partial}{\partial\tau}G_0(\tau)\,d\tau=G(t) \tag{1.17}$$

$$A(t)=\int_0^t G(t-\tau)\frac{\partial}{\partial\tau}\left[\frac{Ba^2(\tau)\,\xi(\tau)}{a^2(\tau)-b^2}-\frac{2a^2(\tau)\,b^2P_0(t)}{a^2(\tau)-b^2}\right]d\tau.$$

Then the appropriate change of the volume

$$l_0(t)=\frac{1}{b^2}\left(\xi(t)-A(t)\right).$$

Taking into account what has been said, we get expressions for the components of deformation tensor and dilatation. Now we calculate $\sigma^{(1)}_{r,\theta},\ e^{(1)}_{r,\theta}$.

Taking into account (1.5) in (1.4) and equating to zero the coefficient λ (of first degree), we get:

$$\int_0^t\left[J_0(t-\tau)+K_0(t-\tau)\right]\frac{\partial^2}{\partial r\partial\tau}l_1(r,\tau)\,d\tau$$
$$+\int_0^t\left[J_1(r,t-\tau)+K_1(r,t-\tau)\right]\frac{\partial^2}{\partial r\partial\tau}l_0(r,\tau)\,d\tau=0.$$

Denote the second summand in the last expression by the known function $F_1(r,t)$ and get

$$\int_0^t\left[J_0(t-\tau)+K_0(t-\tau)\right]\frac{\partial^2}{\partial r\partial\tau}l_1(r,\tau)\,d\tau+\rho F_1^2(r,t)\,r=0 \tag{1.18}$$

$$F_1^2(r,t)=\frac{1}{\rho r}\int_0^t\left[J_1(r,t-\tau)+K_1(r,t-\tau)\right]\frac{\partial^2}{\partial r\partial\tau}l_0(r,\tau)\,d\tau.$$

Comparing (1.18) with (1.5), we find that they are identical, and all subsequent arguments when solving equation (1.18) will be similar to the previous one only by substitution of $F_1^2(r,t)$ for $\omega^2(t)$

Then

$$l_1(r,t)=l_0(t)-\frac{1}{r}\rho r^2\Omega_1(r,t),$$

$$\Omega_1(r,t)=\int_0^t H_0(t-\tau)\frac{\partial}{\partial\tau}F_1^2(r,\tau)\,d\tau.$$

In the similar way, for $l_0(r,t)$we have:

$$l_n(r,t)=l_0(t)-\frac{1}{r}\rho r^2\Omega_n(t),$$

$$\Omega_n(r,t)=\int_0^t H_0(t-\tau)\frac{\partial}{\partial\tau}F_n^2(r,\tau)\,d\tau$$

$$F_n^2(r,t)=\sum_{k=1}^{n}\int_0^t\left[J_k(r,t-\tau)+K_1(r,t-\tau)\right]\frac{\partial^2}{\partial r\partial\tau}l_0(r,\tau)\,d\tau.$$

The boundary conditions for $\sigma_{r,\theta}^{(n)}$ take the form:

$$\sigma_r^{(n)}\left(a\left(t\right),t\right)=0,\sigma_r^{(n)}\left(b,\tau\right)=0 \qquad \left(n=1,2,...\right). \tag{1.19}$$

Here the above stated technique remains valid, but in the expressions (1.14) we should assume $B=0$, $\pi\left(t\right)=0$, and replace the functions $P_1\left(t\right),\ P_0\left(t\right)$ by new known functions $P_{n1}\left(t\right),\ P_{n0}\left(t\right)$.

For simplicity we are restricted only by the calculation of $\sigma_r^{(1)}$

$$\sigma_r^{(1)}=\int_0^t\left[J_0\left(t-\tau\right)+\frac{1}{r}K_0\left(t-\tau\right)\right]\frac{\partial}{\partial\tau}l_0\left(\tau\right)d\tau-\frac{1}{2r^2}\int_0^t K_0\left(t-\tau\right)\frac{\partial}{\partial\tau}A\left(\tau\right)d\tau$$

$$-\frac{1}{r}\rho r^2\left\{F_1^2\left(r,t\right)-\frac{1}{4}\int_0^t K_0\left(t-\tau\right)\frac{\partial\Omega_1\left(r,\tau\right)}{\partial\tau}\partial\tau\right\}.$$

Satisfying boundary conditions (1.19), we get:

$$N_0\left(t\right)-\frac{1}{2a^2\left(t\right)}N\left(t\right)-\frac{\rho a^2\left(t\right)}{2}\left\{F_1^2\left(a\left(t\right),t\right)-\frac{1}{4}\int_0^t K_0\left(t-\tau\right)\frac{\partial\Omega_1\left(a\left(\tau\right),\tau\right)}{\partial\tau}d\tau\right\}=0$$

$$N_0\left(t\right)-\frac{1}{2b^2}N_1\left(t\right)-\frac{\rho b^2}{2}\left\{F_1^2\left(b,t\right)-\frac{1}{4}\int_0^t K_0\left(t-\tau\right)\frac{\partial\Omega_1\left(b,\tau\right)}{\partial\tau}d\tau\right\}=0,$$

where:

$$N_0\left(t\right)=\int_0^t\left[J_0\left(t-\tau\right)+\frac{1}{r}K_0\left(t-\tau\right)\right]\frac{\partial}{\partial\tau}l_0\left(\tau\right)d\tau,$$

$$N_1\left(t\right)=\int_0^t K_0\left(t-\tau\right)\frac{\partial}{\partial\tau}A\left(\tau\right)d\tau$$

$$P_3\left(r,t\right)=\left(\rho r^2/2\right)\left[F_1^2\left(r,t\right)-\frac{1}{4}K_0\left(t-\tau\right)\frac{\partial\Omega_1\left(r,\tau\right)}{\partial\tau}d\tau\right].$$

Excepting N_0, we find:

$$N_1\left(t\right)=-\frac{2a^2\left(t\right)b^2}{a^2\left(t\right)-b^2}\left(P_3\left(a\left(t\right),t\right)-P_3\left(b,t\right)\right).$$

Excepting N_1, we get:

$$N_0\left(t\right)=\frac{1}{2\left(a^2\left(t\right)-b^2\right)}\left(P_3\left(a\left(t\right),t\right)-P_3\left(b,t\right)\right).$$

Allowing for the last expressions, we get:

$$\sigma_r^{(1)}\left(r,t\right)=\frac{1}{2\left(a^2\left(t\right)-b^2\right)}\left(P_3\left(a\left(t\right),t\right)-P_3\left(b,t\right)\right)\cdot\left(1-\frac{2a^2\left(t\right)b^2}{r^2}\right)-\frac{1}{r^2}\rho r^2$$

$$\times\left\{F_1^2\left(r,t\right)-\frac{1}{4}\int_0^t K_0\left(t-\tau\right)\frac{\partial\Omega_1\left(r,t\right)}{\partial\tau}d\tau\right\}.$$

In the same way, we can write $\sigma_\theta^{(1)}\left(r,t\right)$.

As is seen from the last expression $\sigma_r^{(1)}$ $\left(\sigma_\theta^{(1)}\text{ as well}\right)$ are obviously independent of $\xi_0\left(t\right)$, they depend on $\xi_0\left(t\right)$ only by $F_1^2\left(r,t\right)$. This circumstance improves the results of calculations. $\sigma_{r,\theta}^{(1)}$ may be considered as refinement for calculations because of account of temperature dependence of mechanical properties of the material. For $\omega=0$, $t'=t$ (i.e. without temperature dependence of the material properties taken into account) the solution coincides with the solution in [9].

For quality illustration of possibilities of the suggested method, consider some special cases.

1. The material is incompressible: Then $\sigma_{r,\theta}^{0} = -p(t) + \int_0^t K_0(t-\tau)\frac{\partial}{\partial\tau}e_{r,\theta}^{(t)}d\tau, \quad l_0(t) = 0;$ $J(t) \to \infty$ where $p(t)$ is hydrostatic pressure:

$$\xi_0(t) = A(t), \qquad -e_r^0 = e_\theta^0 = A(t)/2r^2.$$

   Allowing fort this, we find:

$$\sigma_{r,\theta}^0 = -p(t) - \frac{1}{2r^2}\int_0^t K_0(t-\tau)\frac{\partial A(\tau)}{\partial\tau}d\tau,$$

$$-p(t) = \frac{1}{2r^2}\left\{\frac{Ba^2(t)A(t)}{a^2(t)-b^2} - \frac{2a^2(t)b^2P_0(t)}{a^2-b^2}\right\},$$

$$BA(t) - \left(\frac{a^2(t)}{b^2}-1\right)\int_0^t K(t-\tau)\frac{\partial}{\partial\tau}A(\tau)d\tau = 2a^2(t)P_1(t).$$

2. Assume that there is no ablation on the internal surface, then $a(t) = a - const$. Integral equation (1.15) is reduced to a convolution type integral:

$$\xi_0(t) - \int_0^t \Phi^*(t-\tau)\xi_0(\tau)d\tau = \pi^*(t),$$

where

$$\Phi^*(t-\tau) = \frac{1}{\Phi_0}\left(1-\frac{a^2}{b^2}\right)\frac{\partial}{\partial\tau}\Phi(t-\tau),$$

$$\pi^*(t) = \left(2a^2/\Phi_0\right)P_1(t), \quad \Phi_0 = B + \Phi(0)\left(1-a^2/b^2\right).$$

   Increase of the internal surface at the expense of ablation may be given in the following form:

$$a^2(0)/a^2(t) = 1 - \left(1-a^2(0)/b^2\right)\frac{t}{t_0}.$$

The pressure influencing on moving internal surface, is given in the form of an exponentially increasing function

$$\pi(t)/k(0) = P_0\left[1-e^{-mt/t_0}\right],$$

where $m, t_0 - const$.

The analytic solution here may be easily obtained by using the Laplace transform [9]. From the above stated we can make the following conclusions:

- We found solutions of a problem on behavior of a hollow viscoelastic cylinder fastened with an elastic shell with uniform pressure and high temperature (i.e. combustion temperature) acting on its internal surface. Mechanical properties of the material depend on temperature that changes the problem into nonlinear one. When ablation on the internal surface is taken into account, a moving boundary nonlinear problem is solved.

- We suggest a method new for solving the stated problem, and this reduces to the solution of a Volterra type integral equation of second kind for $\xi_0(t)$. It was determined that the dependence of the radial shift on $\xi_0(t)$ is linear, and this admits to determine experimentally $u_r(b,t)$ for the series t_k and to find stress distributions not knowing their history of development.

2 Conclusions

The solution of a problem on behavior of rigid viscoelastic cylinders with arbitrary rheology, rigidly built-in interior to an elastic shell in transportations and other operational conditions, is reduced to definition of a function, to the solution of an integral equation. The considered statement has no analognes and is the very adequate approximate-analytic solution of the problem of self-heating of plastic masses under cyclic loadings whose significance of solution was extensively cited in [9]. General scheme of application of a new technique of solution of such problems, variants of the choice of small parameter, expressions for determining other members of the series are given in (1.9) and it is shown that when a small parameter is chosen successfully, account of two members of the series gives favourable practical results.

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